Hints and Solutions JNU 2002

Provided by

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1. α and β are roots of $4x^2 + 3x + 7 = 0$

$$\Rightarrow \quad \alpha + \beta = \frac{-3}{4} \text{ and } \alpha\beta = \frac{7}{4}$$

$$\Rightarrow \quad \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{3}{4}}{\frac{7}{4}}$$

$$\Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-3}{7}$$

2. Every probability distribution possess distribution function but mean, mode and moment generating function may not exist.

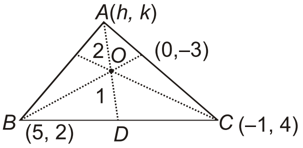
3. $e^{3x+4} = e^4 (e^{3x})$

$$= e^4 \left(1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \ldots \right)$$

so coefficient of x^2 will be

$$e^4 \cdot \frac{3^2}{2!} = \frac{9e^4}{2}$$

4. D is midpoint of BC so, D is



$$\left(\frac{5-1}{2} + \frac{2+4}{2} \right) = (2, 3)$$

As O divides AD in ratio 2 : 1

$$\Rightarrow \quad AO : OD = 2 : 1$$

Let A be (h, k)

$$\Rightarrow \quad \frac{h \times 1 + 2 \times 2}{1 + 2} = 0 \Rightarrow h = -4$$

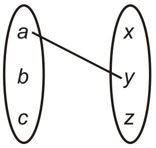
$$\frac{K \times 1 + 2 \times 3}{1 + 2} = -3 \Rightarrow k = -15$$

So A is $(-4, -15)$.

5. $(1+i)^4 \left(1 + \frac{1}{i}\right)^4 = (1+i)^4 \frac{(i+1)^4}{i^4}$

$$= \frac{[(1+i)^2]^4}{1} = (2i)^4 = 16$$

6.



$f(a) = y$ and function is one-one onto,

So, either $f(b) = x$ and $f(c) = z$ or $f(b) = z$ and $f(c) = x$

For the first case $f(c) = z$ and $f^{-1}(x) = b$.

7. R.V.X follows poission distribution if

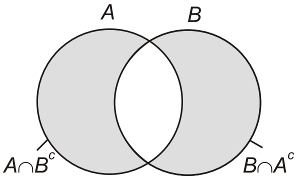
$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}; \; x = 0, 1, 2, \ldots$$

Mean = $E(x) = \lambda$; variance = λ

8. $(\vec{\mathbf{b}} - \vec{\mathbf{c}}) \times (\vec{\mathbf{c}} - \vec{\mathbf{a}}) = \vec{\mathbf{b}} \times \vec{\mathbf{c}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} - \vec{\mathbf{c}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

$$= \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} - 0 + \vec{\mathbf{c}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$$

9.



Probability that exactly one of them occurs

$$= P(A \cap B^c) + P(A^c \cap B)$$
$$= P(A) + P(B) - 2P(A \cap B)$$

11. $|a - b| \geq ||a| - |b||$

12. $p^2 + q^2 = 1;$

$$X = (3p - 4p^3)^2 + (3q - 4q^3)^2$$

$$p = \cos\theta \Rightarrow q = \sin\theta$$

$$\Rightarrow \quad X = (3\cos\theta - 4\cos^8\theta)^2 + (3\sin\theta - 4\sin^3\theta)^2$$

$$\Rightarrow \quad (-\cos 3\theta)^2 + (\sin 3\theta)^2 = \cos^2 3\theta + \sin^2 3\theta = 1$$

13. $A_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}; A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$A_\beta = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$A_\alpha A_\beta = \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{\alpha+\beta}$$

$$A_\alpha \cdot A_{-\alpha} = \begin{bmatrix} \cos(\alpha-\alpha) & \sin(\alpha-\alpha) \\ -\sin(\alpha-\alpha) & \cos(\alpha-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A_\alpha A_\alpha = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} = A_{2\alpha}$$

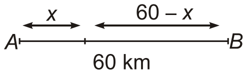
$(A_\alpha)^n = A_{n\alpha}$

$A_\beta A_\alpha = A_\alpha A_\beta$ also.

14. Top must be applied to access an element of a stack.

15. i starts from '0'. If it increases upto 19 then loop will execute 20 times as from 0 to 19 count is 20. Hence, $i < 20$.

17. If x km is the distance of school from A, then total distance covered,



$$d = 150 \times x + 50(60 - x) = 3000 + 100x$$

which is minimum if $x = 0$.

18. $I = \int_0^{\pi/4} \frac{\sin x + \cos x}{25 + 144\sin 2x} dx$

$$= \int_0^{\pi/4} \frac{\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)}{25 + 144\sin 2x} dx$$

$$= \int_0^{\pi/4} \frac{\sqrt{2}\cos x}{25 + 144\cos 2x} dx$$

$$= \sqrt{2}\int_0^{\pi/4} \frac{\cos x}{25 + 144\sin 2x} dx$$

$$= \sqrt{2}\int_0^{\pi/4} \frac{\cos x \ dx}{25 + 144(1 - \sin^2 x)}$$

$$= \sqrt{2}\int_0^{\pi/4} \frac{\cos x}{169 - 288\sin^2 x} dx \ ;$$

Let $\sin x = y \Rightarrow \cos x\, dx = dy$

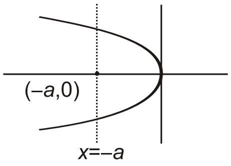
$$\Rightarrow \quad I = \sqrt{2}\int_0^{\pi/4} \frac{dy}{169 - 288\, y^2} = \frac{\sqrt{2}}{288}\int_0^{\frac{1}{\sqrt{2}}} \frac{dy}{\frac{169}{288} - y^2}$$

$$= \frac{1}{144\sqrt{2}}\int_0^{\frac{1}{\sqrt{2}}} \frac{dy}{\left(\frac{13}{12\sqrt{2}}\right) - y^2}$$

$$= \frac{1}{144\sqrt{2}} \cdot \frac{1}{2 \cdot \frac{13}{12\sqrt{2}}} \log \left. \frac{\frac{13}{12\sqrt{2}} + y}{\frac{13}{12\sqrt{2}} - y} \right|_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{2 \cdot 13 \cdot 12} \log 25 = \frac{2\log 5}{2(156)} = \left(\frac{1}{156}\right)\log_e 5$$

19. $y^2 + 4ax = 0 \Rightarrow y^2 = -4ax$



20. (d) None of these

- For heap Binary tree must be complete.
- For Binary search tree left <root> right.
- For complete binary tree if a node has right child it must have left child also.

21. Area of triangle $= \frac{1}{2}\begin{vmatrix} 4 & 6 & 1 \\ x & 4 & 1 \\ 6 & 2 & 1 \end{vmatrix} = 10$

$$\Rightarrow \quad \frac{1}{2}(8 - 4x + 12) = 0$$

$$\Rightarrow \quad 10 - 2x = 10 \Rightarrow x = 0$$

23. Pair of tangents from (4, 3) to the circle $x^2 + y^2 - 2x - 2y = 0$ is

$$SS_1 = T^2$$

$$\Rightarrow \quad (x^2 + y^2 - 2x - 2y)(4^2 + 3^2 - 2 \cdot 4 - 2 \cdot 3)$$
$$= (4x + 3y - x - 4 - y - 3)$$

$$\Rightarrow \quad 11(x^2 + y^2 - 2x - 2y) = (3x + 2y - 7)^2$$

$$\Rightarrow \quad 2x^2 - 12xy + 7y^2 + 20x + 6y - 49 = 0$$

Angle between tangents is θ, then

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \frac{2\sqrt{36 - 14}}{2 + 7} = \frac{2\sqrt{22}}{9}$$

$$\Rightarrow \quad \theta = \tan^{-1}\frac{2\sqrt{22}}{9}$$

24. 10's complement of 3754

$$= 10000 - 3754 = 6246$$

26. $\log 343 = 2.5353$

$\Rightarrow \quad \log 7^3 = 2.5353$

$\Rightarrow \quad 3\log 7 = 2.5353$

$\Rightarrow \quad \log 7 = 0.8451$

Now, $\quad 7^n > 10^5$

$\Rightarrow \quad n\log 7 > 5$

$\Rightarrow \quad n > \frac{5}{\log 7} \Rightarrow n > \frac{5}{0.8451}$

$\Rightarrow \quad n > 5.91$

$\Rightarrow$ n is at least 6.

27. $\cos(\sin^{-1} + 2\cos^{-1} x) = \cos\left(\frac{\pi}{2} + \cos^{-1} x\right)$

$$= -\sin(\cos^{-1} x) = -\sin(\sin^{-1}\sqrt{1 - x^2})$$

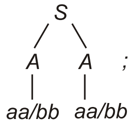
$$= -\sqrt{1 - x^2} = -\sqrt{1 - \left(\frac{1}{3}\right)^2} \text{ at } x = \frac{1}{3}$$

$$= \frac{-2\sqrt{2}}{3}$$

29. (I am outdoors) $\Rightarrow$ 1 am well

$\therefore \quad R \to \sim Q$

30. $S \to AA \ ; A \to aa|bb$



31. $(x - 1)^3 + 8 = 0$

$\Rightarrow \quad (x - 1)^3 = -8$

$\Rightarrow \quad (x - 1)^3 = (-2)^3$

$\Rightarrow \quad x - 1 = -2, -2w, -2w^2$

$\Rightarrow \quad x = 1 - 2, 1 - 2w, 1 - 2w^2 = -1, 1 - 2w, 1 - 2w^2$

32. $y = \log_e x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x} \ ; \frac{d^2y}{dx^2} = -\frac{1}{x^2} \ ; \frac{d^3y}{dx^3} = \frac{2}{x^3} \ ; \frac{d^ny}{dx^n}$$

$$= (-1)^{n-1}\frac{(n-1)!}{x^n} = (-1)^{n-1}(n-1)!\,x^{-n}$$

33. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \quad (\sqrt{2} - 1)\cos\theta = \sin\theta$

$\Rightarrow \quad \cos\theta = \frac{\sin\theta}{\sqrt{2} - 1} = (\sqrt{2} + 1)\sin\theta$ ... (i)

$\cos\theta - \sin\theta = (\sqrt{2} + 1)\sin\theta - \sin\theta = \sqrt{2}\sin\theta$

34. $\int_{-3}^{+3}|x|\,dx = 2\int_0^3 |x|\,dx$ as $|x| = |-x|$;

So, it is even function

$$= 2\int_0^{+3} x\,dx = 2\cdot\left.\frac{x^2}{2}\right|_0^3 = 9$$

36. $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \vec{\mathbf{a}}\times\vec{\mathbf{c}} \Rightarrow \vec{\mathbf{a}}\times(\vec{\mathbf{b}}-\vec{\mathbf{c}}) = 0$

$\Rightarrow$ $\vec{\mathbf{b}}-\vec{\mathbf{c}}$ is parallel to $\vec{\mathbf{a}}$

$$\Rightarrow \quad \vec{\mathbf{b}}-\vec{\mathbf{c}} = \lambda\,\vec{\mathbf{a}}$$

$$\Rightarrow \quad \vec{\mathbf{b}} = \vec{\mathbf{c}} + \lambda\,\vec{\mathbf{a}}$$

38. $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow \quad a(2\cos^2\theta - 1) + b\,(2\sin\theta\cos\theta) = c$$

$$\Rightarrow \quad a(2-\sec^2\theta) + b\,(2\tan\theta) = c\sec^2\theta$$

Multiply be $\sec^2\theta$

$$\Rightarrow \quad a\,(1-\tan^2\theta) + 2b\tan\theta = c\,(1+\tan^2\theta)$$

$$\Rightarrow \quad (a+c)\tan^2\theta - 2b\tan\theta + (c-a) = 0$$

$$\Rightarrow \quad \tan\theta_1 + \tan\theta_2 = \frac{2b}{(a+c)}$$

and $$\tan\theta_1\tan\theta_2 = \frac{(c-a)}{(a+c)}$$

$$\Rightarrow \quad \tan(\theta_1+\theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1\tan\theta_2}$$

$$= \frac{\dfrac{2b}{(a+c)}}{1-\left(\dfrac{c-a}{a+c}\right)} = \frac{2b}{2a} = \frac{b}{a}$$

39. Sum is 10 if $1+9,\ 2+8,\ldots,9+1$ is taken 9 cases.
Sum is 8 if $1+7,\ 2+6,\ldots 7+1$ is taken 7 cases

$$\Rightarrow \quad p_1 = \frac{9}{9c_1\cdot 9c_1};\ p_2 = \frac{7}{9c_1\cdot 9c_1}$$

$$\Rightarrow \quad p_1 + p_2 = \frac{9}{81} + \frac{7}{81} = \frac{16}{81}$$

40. As, 1, 2, 5, 4are adjacent to 3, so 6 is opposite to 3.

41. $\sqrt{p+1} - \sqrt{p-1} = 0$

$\Rightarrow$ $p+1 = p-1 \Rightarrow 1 = -1$ impossible.

42. $^{15}C_7\cdot{}^{8}C_5\cdot{}^{3}C_3 = \dfrac{15!}{7!8!}\cdot\dfrac{8!}{5!3!}\cdot 1 = \dfrac{15!}{7!5!3!}$

43.
$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = \begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix}$$

$$C_1 \leftarrow C_1 + C_2 + C_3$$

$$= (3x+a)\begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix}$$

44. $\sin\theta\cos\theta = \dfrac{\sin 2\theta}{2}$; maximum value of $\sin 2\theta = 1$.

Hence, maximum value of $\sin\theta\cos\theta = \dfrac{1}{2}$

45. Since ( ) enjoys highest priority hence 5*6 will be evaluated first $\Rightarrow$ 30

30 is non-zero hence true.

And (True of Anything) = True $\Rightarrow$ 1

46. $r = r_1 - r_2 - r_3$

$$\Rightarrow \quad \frac{\Delta}{S} = \frac{\Delta}{S-a} - \frac{\Delta}{S-b} - \frac{\Delta}{S-c}$$

$$\Rightarrow \quad \frac{1}{S} = \frac{1}{S-a} - \frac{1}{S-b} - \frac{1}{S-c}$$

$$\Rightarrow \quad \frac{1}{S-b} + \frac{1}{S-c} = \frac{1}{S-a} - \frac{1}{S}$$

$$\Rightarrow \quad \frac{S-c+S-b}{(S-b)(S-c)} = \frac{S-(S-a)}{S\,(S-a)}$$

$$\Rightarrow \quad \frac{a}{(S-b)(S-c)} = \frac{a}{S\,(S-a)}$$

$$\Rightarrow \quad S\,(S-a) = (S-b)(S-c)$$

$$\Rightarrow \quad S^2 - aS = S^2 - (b+c)S + bc$$

$$\Rightarrow \quad (b+c-a)S = bc$$

$$\Rightarrow \quad \frac{(b+c+a)(b+c-a)}{2} = bc$$

$$\Rightarrow \quad (b+c)^2 - a^2 = 2bc$$

$$\Rightarrow \quad b^2 + c^2 = a^2$$

47. $$(1+x^2)\frac{dy}{dx} = (1+y^2)$$

$$\Rightarrow \quad \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both sides of equation (i)

$$\Rightarrow \quad \tan^{-1}y = \tan^{-1}x + \tan^{-1}a$$

$$\Rightarrow \quad \tan^{-1}y - \tan^{-1}x = \tan^{-1}a$$

$$\Rightarrow \quad \tan^{-1}\frac{(y-x)}{1+xy} = \tan^{-1}a$$

$$\Rightarrow \quad \frac{y-x}{1+xy} = a$$

$$\Rightarrow \quad y - x = a\,(1+xy)$$

48. Let (α,β) be any point on locus and (x_1, y_1); (x_2, y_2) be two fixed points, so

$$\frac{\sqrt{(\alpha-x_1)^2 + (\beta-y_1)^2}}{\sqrt{(\alpha-x_2)^2 + (\beta-y_2)^2}} = K$$

$$\Rightarrow \quad (\alpha-x_1)^2 + (\beta-y_1)^2 = K^2\,[(\alpha-x_2)^2 + (\beta-x_2)^2]$$

Coefficient $\alpha^2 = K^2 - 1$ and coefficient of $\beta^2 = K^2 - 1$ also.

So, it is circle.

49.
$$\begin{array}{r} 47 \\ \times\ 52 \\ \hline 116 \\ 303 \\ \hline 3146 \end{array}$$

It is base 8 multiplication.

50.
$$\begin{array}{cccc} S & H & I & P \\ \downarrow & \downarrow & \downarrow & \downarrow \\ V & K & L & S \end{array} \qquad \begin{array}{cccccc} P & E & N & C & I & L \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ S & H & Q & F & L & O \end{array}$$

3 alphabets ahead.

51. If α, β are roots of $px^2 + qx + r = 0$, then

$$\alpha+\beta=\frac{-q}{p};\ \alpha\beta=\frac{r}{p};\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=\frac{q^2}{p^2}-\frac{2r}{p}=\frac{q^2-2pr}{p^2}$$

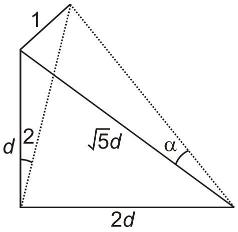
As $\alpha+\beta=\alpha^2+\beta^2$ So $\frac{-q}{p}=\frac{q^2-2pr}{p^2}$

$\Rightarrow$ $-pq=q^2-2pr \Rightarrow q^2=p(2r-q)$

52. After giving value of x increment is done while decrement is done before giving value. (Correct answer is $x = 4$, $y = 4$). (So none is correct)

53. Testing may be done for logical as well as syntax error, debugging is the process to correct those errors, testing and debugging may be separate process.

54. Let l be the length of tree and d be the width of the river, So $\tan\alpha=\frac{1}{\sqrt{5}d}$ and $\tan 2\alpha=\frac{1}{d}$



$\Rightarrow$ $\frac{\tan 2\alpha}{\tan\alpha}=\sqrt{5}\Rightarrow\frac{2\tan\alpha}{1-\tan^2\alpha}=\sqrt{5}\tan\alpha$

$\Rightarrow$ $\sqrt{5}\tan^2\alpha=\sqrt{5}-2\Rightarrow\tan^2\alpha=\frac{\sqrt{5}-2}{\sqrt{5}}$

$\Rightarrow$ $\tan\alpha=\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}}}\Rightarrow\alpha=\frac{\pi}{12}$

55. $\log 2, \log(2^x-1), \log(2^x+3)$ are in AP.

So, $2\log(2^x-1)=\log 2+\log(2^x+3)$

$\Rightarrow$ $(2^x-1)^2=2(2^x+3)$

$\Rightarrow$ $(y-1)^2=2(y+3)$

$\Rightarrow$ $y^2-4y-5=0$; $y=2^x$

$\Rightarrow$ $(y-5)(y+1)=0$

$\Rightarrow$ $y=5, -1\Rightarrow y=5$ as $y\neq -1$,

So, $2^x=5\Rightarrow x=\log_2 5$

56. $a*\left(\frac{(c+d)}{a}\right)\sim a*\left(\frac{(cd+a)}{a}\right)\sim a*(cd+a/)\sim acd+a/*$

57. $|x|=x$ if $x\geq 0$

$=-x$ if $x\leq 0$

$\Rightarrow$ $f(x)=0$ if $x\geq 0$

$=2$ if $x<0$

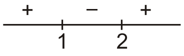
At $x=0$, Left hand limit $=2$

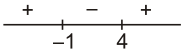
Right hand limit $=0$

$f(x)$ is continuous everywhere except at $x=0$.

58. $x^2-3x+2>0$ $\quad$ $x^2-3x-4\leq 0$

$\Rightarrow(x-1)(x-2)>0$ $\quad$ $(x-4)(x+1)\leq 0$





$\Rightarrow$ $x\in[-1,1)\cup(2,4]$

60. $f(x)=x^2-2x+2 \Rightarrow f'(x)=2x-2=0$

$\Rightarrow$ $x=1$

$f''(x)=2>0$

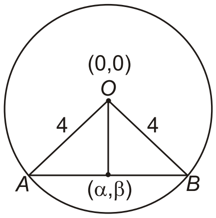
$\Rightarrow$ $x=1$ is a minima.

61. $x=\frac{a(1-t)^2}{1+t^2}=\frac{a[(1+t^2)-2t]}{(1+t^2)}=a-\frac{a}{b}\left(\frac{2bt}{1+t^2}\right)$

$\Rightarrow$ $x=a-\frac{ay}{b}\Rightarrow 1=0-\frac{a}{b}\frac{dy}{dx}$

$\Rightarrow$ $\frac{dy}{dx}=-\frac{b}{a}$

62.



Let (α,β) be the midpoint of chord.

$$AB=4\sqrt{2}$$

Distance between (0, 0) and $(\alpha,\beta)=\sqrt{\alpha^2+\beta^2}$

Area of triangle $OAB=\frac{1}{2}\cdot 4\cdot 4=\frac{1}{2}\cdot\sqrt{\alpha^2+\beta^2}\cdot 4\sqrt{2}$

$\Rightarrow$ $\alpha^2+\beta^2=8$

$\Rightarrow$ $x^2+y^2=8$

63. $x-\frac{2}{(x-1)}=1-\frac{2}{(x-1)}$

$x=1$ is not a solution as denominator becomes 0.

64. $\left.\begin{array}{r}3x+y+2z=3\\2x-3y-z=-3\\x+2y+z=4\end{array}\right]$

$\Rightarrow$ $A:B=\left[\begin{array}{ccc|c}1&2&1&4\\2&-3&-1&-3\\3&1&2&3\end{array}\right]$

$=\left[\begin{array}{ccc|c}1&2&1&4\\0&-7&-3&-11\\0&-5&-1&9\end{array}\right]$

$\Rightarrow$ rank $(A)=$ rank $(A:B)=3$

$\Rightarrow$ unique solution.

65. $\lim\limits_{x\to 0}\frac{|\sin x|}{x}$; LHL $=\lim\limits_{x\to 0}\frac{-\sin x}{x}=-1$

RHL $=\lim\limits_{x\to 0}\frac{\sin x}{x}=1$; LHL $\neq$ RHL

$\Rightarrow$ Limit does not exist.

66. Consecutive prime numbers are taken.

67. $x+y=\sin(x+y)$

$\Rightarrow$ $1+\frac{dy}{dx}=\cos(x+y)\left[1+\frac{dy}{dx}\right]$

$\Rightarrow$ $\frac{dy}{dx}[\cos(x+y)-1]=[1-\cos(x+y)]$

$\Rightarrow$ $\frac{dy}{dx}=-1$

68. $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a, b > \Rightarrow -$ve real part.

69. Volume of paralleopiped for three adjacent sides

$$\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}} = 0$$

$\Rightarrow$ $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = 0$; as they are planar.

70. If year starts with Sunday it will end on Sunday, but year can start on anyone of 7 days.

Hence probability $= \dfrac{1}{7}$

71. $$\frac{\sin(x+y)}{\sin(x-y)} = \frac{p+q}{p-q}$$

$\Rightarrow$ $$\frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{2p}{2q}$$

$\Rightarrow$ $$\frac{2\sin x \cos y}{2\cos x \sin y} = \frac{p}{q}$$

$\Rightarrow$ $$\frac{\tan x}{\tan y} = \frac{p}{q}$$

By componendo and dividendo.

72. $80 - \dfrac{(84-48)}{100}$ will be exact mean $= 80 - (0.36) = 79.64$.

73. Coefficient of rth term in $(x+y)^n = n_{c_{r-1}}$

$\Rightarrow$ $n_{c_3} = n_{c_{12}} \Rightarrow n = 3 + 12 = 15$

74. $(a+b+c)^2 \geq 0$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow$ $ab + bc + ca \geq -\dfrac{(a^2 + b^2 + c^2)}{2} \geq -\dfrac{1}{2}$

77. $x = (*P++) + 25;$

Since, the operators * and ++ have same hirarchy and the associativity of these operators is right to left hence ++ operator will be operated on *P* and since it is syfix to *P*, will operate later hence **P ie* 34 + 25 $\Rightarrow$ 59 will be assigned to *x* the *P* will point to next element of the array.

78. 1 rectangle is replaced by 1 ellipse and then vice-versa is done.

79. $x^2 + y^2 - 14x$ at (6, –7) is

$$36 + 49 - 81 = 1 > 0$$

$\Rightarrow$ outside circle.

80. After n years population will be

$$N\left(1 + \frac{5}{100}\right)^n = 2N \Rightarrow \left(1 + \frac{5}{100}\right)^n = 2$$

$\Rightarrow$ $$n = \frac{(\log 2)}{\log(1.05)} = 20 \log 2$$

81. $a^2 = b^2 \,\&\, b^2 = a^2(e^2 - 1)$

$\Rightarrow$ $e^2 = 2$

$\Rightarrow$ $e = \sqrt{2}$

82. $$\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix}$$

$\Rightarrow$ $(2-y)(y^2 - 15y + 26) - 2(2 - 2y) + 3(-7 + 3y) = 0$

$\Rightarrow$ $-y^3 + 17y^2 - 43y + 27 = 0$

On putting $y = 1$ to get

$$\text{LHS} = -1 + 17 - 43 + 27 = 0$$

83. $y = \dfrac{1}{\log 10(1-x)} + \sqrt{x+2}$

Domain consist for values of x for which y is real

Here $x + 2 \geq 0 \Rightarrow x \geq -2$ ...(i)

$(1-x) > 0 \Rightarrow 1 > x$ ...(ii)

and $1 - x \neq 1 \Rightarrow x \neq 0$ ...(iii)

$\Rightarrow$ [–2, 1] excluding 0.

84. Let $z = x + iy\,;|z| = z + 1 + 2i$

$\Rightarrow$ $\sqrt{x^2 + y^2} = x + iy + 1 + 2i = (x+1) + i(y+2)$

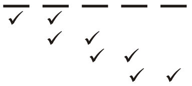
Equating real and imaginary parts of both sides

$$\sqrt{x^2 + y^2} = x + 1 \text{ and } y + 2 = 0 \Rightarrow y = -2$$

$$= x^2 + 4 = (x+1)^2 \Rightarrow 2x + 1 = 4 \Rightarrow x = \frac{3}{2}$$

So, $$z = \left(\frac{3}{2}\right) - 2i$$

85.



Two consecutive head can appear in 4 ways.

Remaining places goes to tail, so required probability is

$$^4C_1\left(\frac{1}{4}\right)^2\left(1 - \frac{1}{4}\right)^3 = \frac{4 \times 3^3}{4^5} = \frac{3^3}{4^4}$$

86. As $AH = G^2\,; H$ being harmonic mean.

$$2A + G^2 = 27$$

$\Rightarrow$ $$2A + AH = 27$$

$\Rightarrow$ $$6A = 27\,; H = 4;$$

$\Rightarrow$ $$A = \frac{9}{2}$$

If x and y are numbers, then

$$A = \frac{x+y}{2}\,; H = \frac{2xy}{(x+y)}$$

$\Rightarrow$ $$x + y = 9\,; xy = 18$$

$\Rightarrow$ $$x - y = 3$$

$\Rightarrow$ $$x = 6, y = 3$$

89. $f(x) = (e^{ix})(e^{i3x}) \ldots\ldots (e^{i(2n-1)x})$

$$= e^{ix(1+3\ldots\ldots(2n-1)x}) = e^{in^2}$$

$$f''(x) = (in^2)^2 e^{in^2 x} = -n^4 f(x)$$

90. For ellipse $b^2 = a^2(1 - e^2) \Rightarrow \dfrac{b^2}{a^2} = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

Two diameters are conjugate if

$$m_1 m_2 = -\frac{b^2}{a^2}$$

$\Rightarrow$ $$m_1\left(-\frac{2}{3}\right) = -\frac{2}{3} \Rightarrow m_1 = 1$$

So, $y = x$ is the required diameter.

91. $f(x) = x + e^x$

$\Rightarrow$ $$f'(x) = 1 > 0$$

Increasing and continuous function

$$f(0) = 0 + e^0 = 1$$

$$f(-\infty) = -\infty\,;\, f(\infty) = \infty$$

It has one root between $-\infty$ and 0 as $f(-\infty)$ and $f(0)$ are of opposite sign.

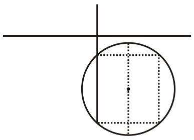
92. Let $x = \tan\theta \Rightarrow \dfrac{1-x^2}{1+x^2} = \cos 2\theta = \sin\left(\dfrac{\pi}{2} - 2\theta\right)$

and $\dfrac{2x}{1+x^2} = \dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

We have to differentiate $\dfrac{\pi}{2} - 2\theta$ w.r.t. 2θ

$$= \frac{\dfrac{d}{d\theta}\left(\dfrac{\pi}{2} - 2\theta\right)}{\dfrac{d}{d\theta}(2\theta)} = \frac{-2}{2} = -1$$

93. $x^2 + y^2 - 2x + 4y + 3 = 0$



$\Rightarrow \quad (x-1)^2 + (y+2)^2 = (\sqrt{2})^2$

Diameter of circle $= 2\sqrt{2}$

So, diagonal of square $= 2\sqrt{2}$

So, side of square = 2

So, vertices of square $= (1 \pm 1, -2 \pm 1)$

$= (0,1), (0,-3), (2,-1), (2,-3)$

94. $\left(\dfrac{9}{15}\right)^7 - \left(\dfrac{8}{15}\right)^7$; as numbers from 1 to 9 is selected 7 times ans we subtract the case in which number 1 to 8 is selected all 7 times.

95.

3, 9, 20, 38, 65, 103, 154

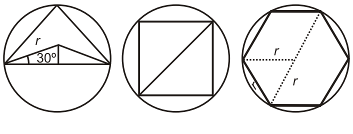
↓ ↓ ↓ ↓ ↓ ↓

6 11 18 27 38 51

↓ ↓ ↓ ↓ ↓

5 7 9 11 13

96.



Let r be the radius of circle and a, b, c be the length of sides of triangle, square and hexagon.

$$a = 2 \cdot r \cdot \frac{\sqrt{3}}{2} = r\sqrt{3}\,;\; b = r\sqrt{2}\,;\; c = r$$

97. $f(x) = x^3 - 2x - 5$

$\Rightarrow \quad f'(x) = 3x^2 - 2$

As $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$$

$$= \frac{2x_n^3 + 5}{3x_n^2 - 2}\,; n = 0, 1, 2.....$$

Let $x_0 = 2$

$\Rightarrow \quad x_1 = \dfrac{16+5}{12-2} = 2.1$

$$x_2 = \frac{2(2.1)^3 + 5}{3(2.1)^2 - 2} = \frac{18.522 + 5}{13.23 - 2} = \frac{23.522}{11.23} = 2.09$$

98. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cup B) \le P(A) + P(B)$

100. $(a, a) \in R,\ \forall\, a \in A$ Hence Reflexive

$(a, b) \wedge (b, c) \in R \Rightarrow (a, c) \in R$; hence transitive

$(6, 12) \in R$ but $(12, 6) \in R$, so it is not symmetric.

101. You can not initialize extern variable.

ie extern int i = 5;

produces an error.

102. $y = 2x + 1$ and $x = 4 \Rightarrow (4, 9)$ is point of intersection

$y = 3x + 1$ and $x = 4 \Rightarrow (4, 13)$

$y = 2x + 1$ and $y = 3x + 1 \Rightarrow (0, 1)$

Area of triangle with vertices (4, 9), (4, 13) and (0, 1) is

$$\frac{1}{2}\begin{vmatrix} 4 & 9 & 1 \\ 4 & 13 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 8$$

103. $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ Sin^2 C & \cot C & 1 \end{vmatrix}$

$$= \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B - \sin^2 A & \cot B - \cot A & 0 \\ \sin^2 c - \sin^2 A & \cot C - \cot A & 0 \end{vmatrix}$$

$$R_2 \leftarrow R_2 - R_1; R_3 \leftarrow R_3 - R_1$$

$$= (\sin^2 B - \sin^2 A)(\cot C - \cot A) - (\sin^2 C - \sin^2 A)$$
$$(\cot B - \cot A)$$

If $A = B = C = \dfrac{\pi}{3}$ above becomes $\underline{0}$.

104. $(1 + x^n) = n_{c_0} + n_{c_1}x + n_{c_2}x^2 + ...+ n_{c_n}x^n$

On putting x = 4 on both sides, we get

$$5^n = 1 + 4(n_{c_1} + 4n_{c_2} + ...+ 4^{n-1})$$

$$\Rightarrow \quad 4(n_{c_1} + 4n_{c_2} + ...+ 4n^{n-1}) = 5^n - 1$$

105. $\det[A - \lambda I] = 0$ gives eigen values.

107.

i/0	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
⋮												
i	0		1			2		3 .... j				
9												

Row number i means we have completed 0 to $i - 1$ *ie* i rows and each row contain 12 columns *ie* 12 * i items are there upto $i - 1$ row. In the ith row j means that it is $(j+1)^{\text{th}}$ element because elements starts from 0. Hence, it is element number $12 \times i + j + 1$. Hence, (b) is the correct option.

109 $I = \int_0^{\pi} \frac{\sin(n+m)x + \sin(n-m)x}{2} dx$

$$I = \int_0^{\frac{\pi}{2}} \{\sin(n+m)x + \sin(n-m)x\} dx$$

$$= \left. \frac{-\cos(n+m)x}{n+m} - \frac{\cos(n-m)x}{(n-m)} \right|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{n+m} + \frac{1}{n-m} = \frac{2n}{n^2 - m^2}$$

110. $n-1_{c_3} + n-1_{c_4} > n_{c_3}$

$\Rightarrow \quad n-1+1_{c_4} > n_{c_3}$

$\Rightarrow \quad n_{c_4} > n_{c_3}$

$\Rightarrow \quad n > 4+3 \Rightarrow n > 7$

111. $\tan\alpha = \frac{m}{m+1}; \tan\beta = \frac{1}{2m+1}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{(m+1)(2m+1)}}$$

$$= \frac{m(2m+1) + m + 1}{(2m+1)(m+1) - m} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$$= \tan\frac{\pi}{4}$$

$\Rightarrow \quad \alpha + \beta = \frac{\pi}{4}.$

112. Mean $= np = \frac{15}{4}$;

Variance $= npq = \frac{15}{16}$

$$\frac{npq}{np} = \frac{\frac{15}{16}}{\frac{15}{4}} \Rightarrow q = \frac{1}{4}$$

$\Rightarrow \quad p = 1 - q = 1 - \frac{1}{4} = \frac{3}{4}$

113. $\alpha, \alpha+1$ are roots of $x^2 - bx + c = 0$, so $\alpha + \alpha + 1 = b$

$\Rightarrow \quad 2\alpha + 1 = b$

$$\alpha(\alpha+1) = c$$

$$b^2 - 4c = (4\alpha^2 + 4\alpha + 1) - 4(\alpha^2 + \alpha) = 1$$

114. $A = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 2a+1 & 2b+1 & 2c+1 \\ -2a+1 & -2b+1 & -2c+1 \end{vmatrix}$$

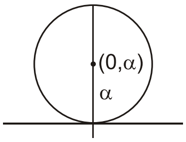
$$R_2 \leftarrow R_2 - R_1 i \, R_3 \leftarrow R_3 - R_1$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 2a+1 & 2b+1 & 2c+1 \\ 2 & 2 & 2 \end{vmatrix} \qquad R_3 \leftarrow R_3 + R_2$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 2a & 2b & 2c \\ 2 & 2 & 2 \end{vmatrix} \qquad R_2 \leftarrow R_2 - \frac{1}{2} R_3$$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = 4B.$$

115.



$$x^2 + (y-\alpha)^2 = \alpha^2 \qquad \text{...(i)}$$

$\Rightarrow \quad 2x + 2(y-\alpha)\frac{dy}{dx} = 0$

$\Rightarrow \quad (y-\alpha) = \frac{-x}{\left(\frac{dy}{dx}\right)}$ and $\alpha = y + \frac{x}{\left(\frac{dy}{dx}\right)}$

On putting in Eq. (i), we get

$\Rightarrow \quad x^2 + \left(\frac{-x}{\frac{dy}{dx}}\right)^2 = \left(y + \frac{x}{\frac{dy}{dx}}\right)^2$

$\Rightarrow \quad x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = y^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} + \frac{2xy}{\frac{dy}{dx}}$

$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} = 2xy$

116. First of all incr() is called. In incr() we have a static variable i (Default initial value of static value is zero) and in the printf statement pre increment operator incremented i to 1 and prints 1.

Next decr() is being called. In decr we have another static variable i and in the printf statement due to post decrement operator it prints 0 and then decrements i to – 1.

Next when incr() is called again we get the old value *ie* 1. Pre increment operator increments i to 2 and prints 2.

117. $A - (B - C) = A - (B \cap C')$

$$= A \cap (B \cap C')' = A \cap (B' \cup C)$$

$$= (A \cap B') \cup (B' \cup C)$$

$$= (A - B) \cup (A \cap C)$$

119. $S = 1^2 + 2.2^2 + 3^2 + 2.4^2 + \ldots . n$ terms

If $n = 2m + 1$ *ie*, odd.

$S = 1^2 + 3^2 + \ldots ..(2m+1)^2 + 2(2^2 + 4^2 + \ldots .. + (2m)^2)$

$$= 1^2 + 2^2 + 3^2 + \ldots .. + (2m+1)^2 + 2^2(1^2 + 2^2 + \ldots .. + m^2)$$

$$= \frac{(2m+1)(2m+2)(4m+3)}{6} + 4\frac{m(m+1)(2m+1)}{6}$$

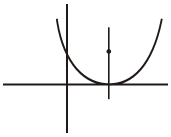
$$= \frac{2(m+1)}{6}[8m^2 + 10m + 3 + 4m^2 + 2m]$$

$$= \frac{(m+1)}{3}(12m^2 + 12m + 3)$$

$$= (m+1)(4m^2 + 4m + 1) = (m+1)(2m+1)^2$$

$$= n^2\left(\frac{n-1}{2} + 1\right) = \frac{n^2(n+1)}{2}$$

120.



$$(x-1)^2 - 8y = 0$$

$$\Rightarrow \qquad 8y = (x-1)^2$$

$$\Rightarrow \qquad (x-1)^2 = 8y$$

Vertex of parabola is (1, 0)

Focus of parabola is (1, 2)

Radius of circle = distance between (1, 0) and (1, 2)

= 2 unit.

Circle has centre at (1, 2)

Hence circle is $(x-1)^2 + (y-2)^2 = (2)^2$

$$\Rightarrow \qquad x^2 + y^2 - 2x - 4y + 1 = 0$$

Answers

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(c)	**7.**	(a)	**8.**	(d)	**9.**	(a)	**10.**	(b)
11.	(c)	**12.**	(a)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(d)	**20.**	(b)
21.	(a)	**22.**	(d)	**23.**	(d)	**24.**	(b)	**25.**	(b)	**26.**	(d)	**27.**	(d)	**28.**	(d)	**29.**	(d)	**30.**	(c)
31.	(b)	**32.**	(d)	**33.**	(a)	**34.**	(b)	**35.**	(a)	**36.**	(a)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(b)
41.	(d)	**42.**	(a)	**43.**	(c)	**44.**	(d)	**45.**	(c)	**46.**	(d)	**47.**	(c)	**48.**	(b)	**49.**	(d)	**50.**	(b)
51.	(d)	**52.**	(b)	**53.**	(d)	**54.**	(b)	**55.**	(a)	**56.**	(d)	**57.**	(b)	**58.**	(c)	**59.**	(b)	**60.**	(c)
61.	(d)	**62.**	(a)	**63.**	(a)	**64.**	(c)	**65.**	(d)	**66.**	(c)	**67.**	(b)	**68.**	(c)	**69.**	(a)	**70.**	(d)
71.	(b)	**72.**	(c)	**73.**	(a)	**74.**	(b)	**75.**	(b)	**76.**	(c)	**77.**	(b)	**78.**	(d)	**79.**	(c)	**80.**	(b)
81.	(b)	**82.**	(b)	**83.**	(c)	**84.**	(a)	**85.**	(d)	**86.**	(a)	**87.**	(b)	**88.**	(a)	**89.**	(b)	**90.**	(d)
91.	(c)	**92.**	(a)	**93.**	(d)	**94.**	(d)	**95.**	(a)	**96.**	(d)	**97.**	(b)	**98.**	(d)	**99.**	(c)	**100.**	(d)
101.	(b)	**102.**	(d)	**103.**	(a)	**104.**	(d)	**105.**	(b)	**106.**	(ab)	**107.**	(b)	**108.**	(c)	**109.**	(a)	**110.**	(c)
111.	(d)	**112.**	(d)	**113.**	(b)	**114.**	(a)	**115.**	(b)	**116.**	(c)	**117.**	(a)	**118.**	(b)	**119.**	(b)	**120.**	(a)