MATHEMATICS - 1998

PART - A

Directions : *Read questions 1 to 40 carefully and choose from amongst the alternatives given below each question the correct lettered choice(s). A question may have ONE OR MORE correct alternatives. In order to secure any marks for a given question, ALL correct lettered alternative(s) must be chosen.*

1. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals :

(A) $128\,\omega$ (B) $-128\,\omega$
(C) $128\,\omega^2$ (D) $-128\,\omega^2$.

2. Let T_r be the rth term of an A.P., for $r = 1, 2, 3, \ldots\ldots$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$
(C) 1 (D) 0

3. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is :

(A) at least 30 (B) at most 20
(C) exactly 25 (D) none of the above

4. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a :

(A) rectangle (B) square
(C) cyclic quadrilateral (D) rhombus.

5. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is :

(A) 0 (B) 1
(C) 3 (D) 4

6. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is :

(A) 1 (B) 2
(C) 0 (D) $\frac{1}{2}$

7. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals :

(A) 8 (B) 6
(C) 10 (D) 12

8. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then :

(A) $a = 2, b = 4$ (B) $a = 3, b = 4$

(C) $a = 2, b = 3$ (D) $a = 3, b = 5$

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then :

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$

(C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

10. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :

(A) $\frac{13}{32}$ (B) $\frac{1}{4}$

(C) $\frac{1}{32}$ (D) $\frac{3}{16}$

11. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals :

(A) i (B) $i - 1$

(C) $-i$ (D) 0

12. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2x})$ attains its maximum is :

(A) 0 (B) 1

(C) 2 (D) infinite

13. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f :

(A) does not exist because f is unbounded.

(B) is not attained even though f is bounded

(C) is equal to 1

(D) is equal to -1

14. Number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240 is :

(A) 4 (B) 8

(C) 10 (D) 3

15. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$:

(A) exists and it equals $\sqrt{2}$

(B) exists and it equals $-\sqrt{2}$

(C) does not exist because $x - 1 \to 0$

(D) does not exist because left hand limit is not equal to right hand limit

16. If in a triangle PQR, $\sin P$, $\sin Q$, $\sin R$ are in A. P., then :

(A) the altitudes are in A. P. (B) the altitudes are in H. P.
(C) the medians are in G. P. (D) the medians are in A. P.

17. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals :

(A) $(n-1)a_n$ (B) na_n
(C) $\frac{1}{2}na_n$ (D) None of the above

18. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational point(s).

(A) centroid (B) incentre
(C) circumcentre (D) orthocentre

(A rational point is a point both of whose co-ordinates are rational numbers)

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is :

(A) 0 (B) 1
(C) 2 (D) infinite.

20. If $x > 1$, $y > 1$, $z > 1$ are in G. P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in :

(A) A.P. (B) H.P.
(C) G.P. (D) None of the above

21. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is :

(A) 0 (B) 5
(C) 6 (D) 10

22. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is :

(A) 5 (B) 4
(C) 3 (D) 2

23. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then :

(A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ (B) $f(x) = \sin x$, $g(x) = |x|$
(C) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$ (D) f and g cannot be determined

24. Let $A_0 A_1 A_2 A_3 A_4 A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments $A_0 A_1$, $A_0 A_2$ and $A_0 A_4$ is :

(A) $\frac{3}{4}$ (B) $3\sqrt{3}$
(C) 3 (D) $\frac{3\sqrt{3}}{2}$

25. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) \cdot \vec{u}$

(C) $\vec{v} \cdot (\vec{u} \times \vec{w})$ I(D) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

26. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is :

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

27. Let $h(x) = \min\{x, x^2\}$, for every real number of x. Then :

(A) h is continuous for all x

(B) h is differentiable for all x

(C) $h'(x) = 1$, for all $x > 1$

(D) h is not differentiable at two values of x

28. If $f(x) = 3x - 5$, then $f^{-1}(x)$:

(A) is given by $\frac{1}{3x-5}$

(B) is given by $\frac{x+5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto.

29. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then.

(A) $P(E/F) + P(\overline{E}/F) = 1$ (B) $P(E/F) + P(E/\overline{F}) = 1$

(C) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (D) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

30. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then :

(A) $x = 3, y = 1$ (B) $x = 1, y = 3$

(C) $x = 0, y = 3$ (D) $x = 0, y = 0$

31. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :

(A) $\frac{1}{2}$ (B) $\frac{1}{32}$

(C) $\frac{31}{32}$ (D) $\frac{1}{5}$

32. An n – digit number is a positive number with exactly n digits. Nine hundred distinct n –digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is :

(A) 6 (B) 7
(C) 8 (D) 9

33. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals :

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$
(C) $\frac{2}{15}$ (D) $\frac{1}{3}$

34. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then :

(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$
(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 - 3n + 3$

35. Which of the following number(s) is/are rational ?

(A) $\sin 15°$ (B) $\cos 15°$
(C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

36. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then :

(A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$
(C) $x_1 x_2 x_3 x_4 = c^4$ (D) $y_1 y_2 y_3 y_4 = c^4$

37. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then :

(A) occurrence of $E \Rightarrow$ occurrence of F
(B) occurrence of $F \Rightarrow$ occurrence of E
(C) non-occurrence of E $\Rightarrow$ non-occurrence of F
(D) none of the above implications holds

38. Which of the following expressions are meaningful question

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$
(C) $(\vec{u} \cdot \vec{v})\, \vec{w}$ (D) $\vec{u} \times (\vec{v} \cdot \vec{w})$

39. If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is :

(A) $\frac{1}{2}$ (B) 0
(C) 1 (D) $-\frac{1}{2}$

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then :

(A) h is increasing whenever f is increasing

(B) h is increasing whenever f is decreasing

(C) h is decreasing whenever f is decreasing

(D) nothing can be said in general.

ANSWERS

1. (D)	**2.** (C)	**3.** (C)	**4.** (D)	**5.** (B)	**6.** (A)
7. (C)	**8.** (C)	**9.** (D)	**10.** (A)	**11.** (B)	**12.** (A)
13. (D)	**14.** (A)	**15.** (D)	**16** (B)	**17.** (C)	**18.** (A)
19. (C)	**20.** (B)	**21.** (C)	**22.** (C)	**23.** (A)	**24.** (C)
25. (C)	**26.** (B)	**27.** (A), (C), (D)	**28.** (B)	**29.** (A), (D)	**30.** (D)
31. (A)	**32.** (B)	**33.** (B)	**34.** (B)	**35.** (C)	
36. (A), (B), (C), (D)		**37.** (D)	**38.** (A), (C)	**39.** (A)	**40.** (A), (C)

SOLUTIONS

1. $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$$= (-2\omega^2)^7 = (-2)^7 (\omega^2)^7 = -128 \cdot \omega^{14} = -128\,\omega^2$$

Therefore, (D) is the Ans.

2. Let $T_m = a + (m-1)\,d = \dfrac{1}{n}$

and $T_n = a + (n-1)\,d = \dfrac{1}{m}$

$\Rightarrow \quad (m-n)\,d = \dfrac{1}{n} - \dfrac{1}{m} = \dfrac{m-n}{mn} \quad \Rightarrow \quad d = \dfrac{1}{mn}$

Again $T_{mn} = a + (mn-1)\,d$

$$= a + (mn - n + n - 1)\,d$$

$$= a + (n-1)\,d + (mn-n)\,d$$

$$= T_n + n(m-1) \cdot \frac{1}{mn}$$

$$= \frac{1}{m} + \frac{(m-1)}{m} = \frac{1}{m} + 1 - \frac{1}{m} = 1$$

Therefore, (C) is the Ans.

3. Let number of newspaper which are read be n. Then

$$60n = 300 \times 5$$

$\Rightarrow \quad n = 25$

Therefore, (C) is the Ans.

4. Slope of $x + 3y = 4$ is $-1/3$

and slope of $6x - 2y = 7$ is 3.