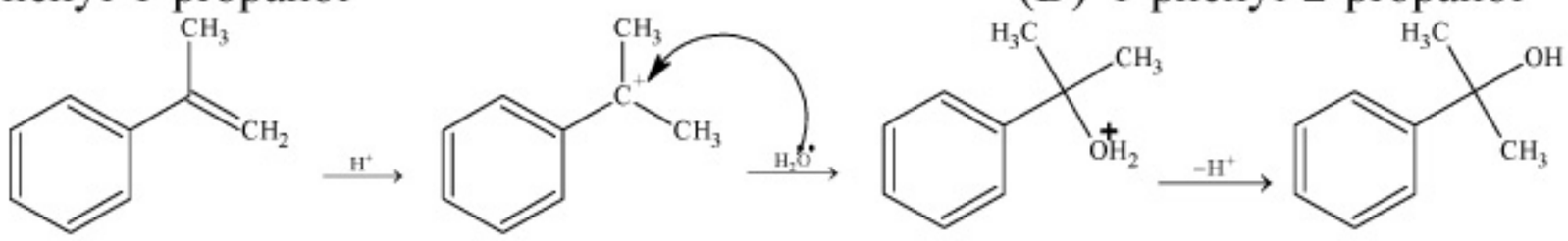
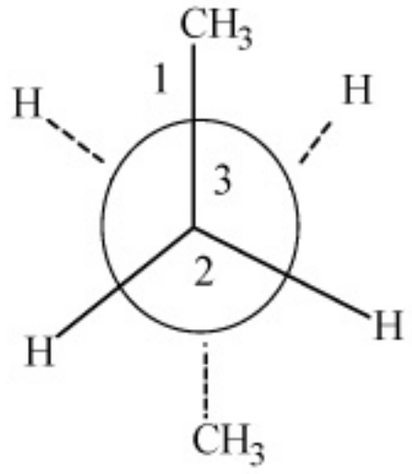
IIT–JEE, 2004 Screening

1*. 2- phenyl propene on acidic hydration gives
(A) 2-phenyl-2-propanol
(B) 2-phenyl-1-propanol
(C) 3-phenyl-1-propanol
(D) 1-phenyl-2-propanol

Ans. (A)

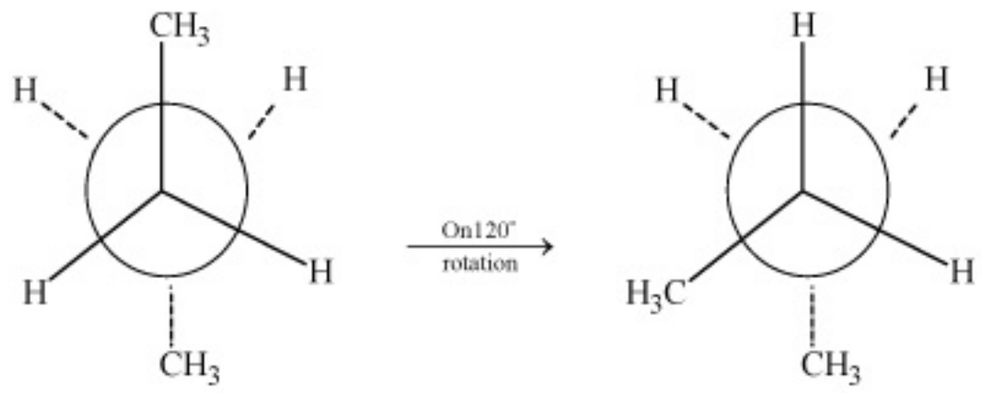


2*.



C_2 is rotated anticlockwise 120°about C_2- C_3 bond. The resulting conformer is
(A) Partially eclipsed
(B) Eclipsed
(C) Gauche
(D) Staggered

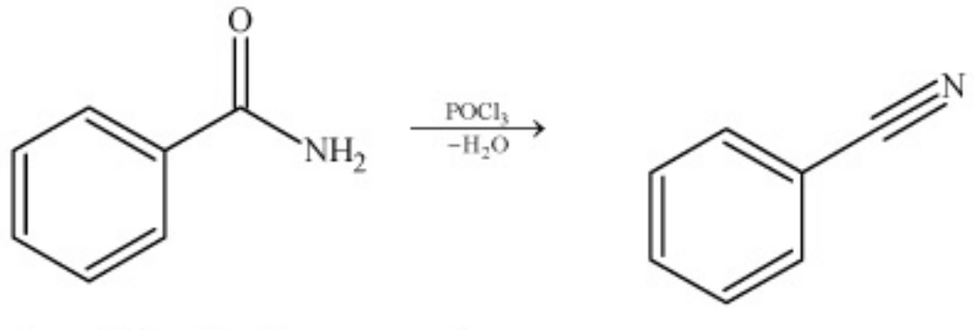
Ans. (C)



The resulting conformer is a Gauche conformer

3. Benzamide on treatment with $POCl_3$ gives
(A) Aniline
(B) Benzonitrile
(C) Chlorobenzene
(D) Benzyl amine

Ans. (B)

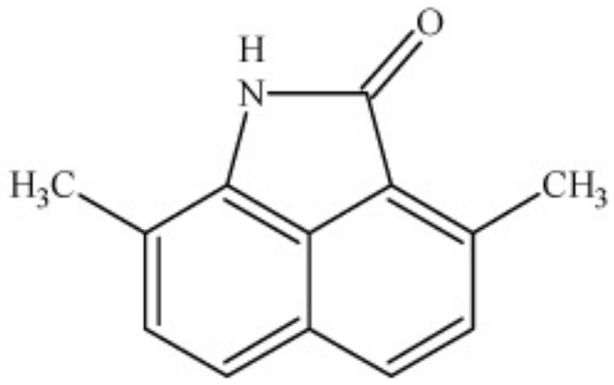


$POCl_3$ is a dehydrating agent

4. The methods chiefly used for the extraction of Lead & Tin from their ores are respectively
(A) Self reduction & carbon reduction
(B) Self reduction & electrolytic reduction
(C) Carbon reduction & self reduction
(D) Cyanide process & carbon reduction

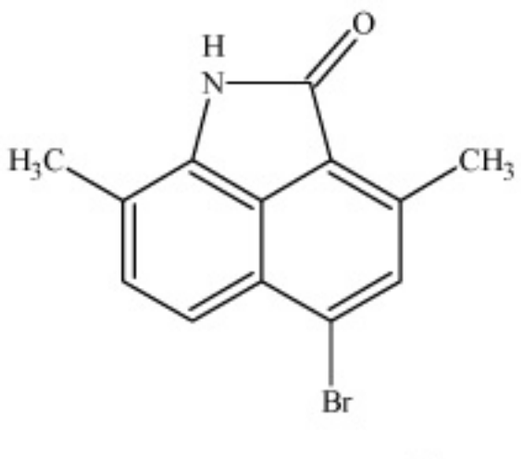
Ans (A) Factual

5*.

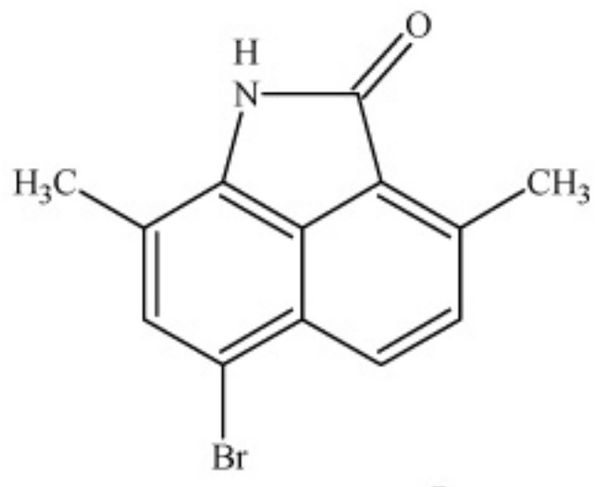


$\xrightarrow{Fe/Br_2}$ Product on monobromination of this compound is

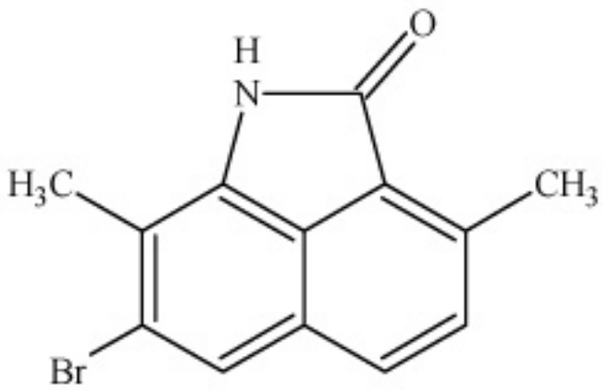
(A)



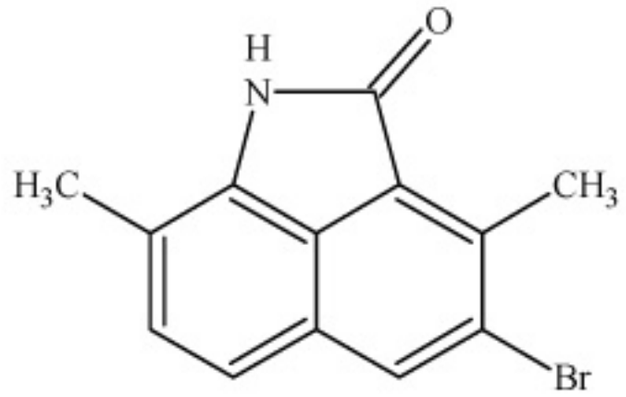
(B)



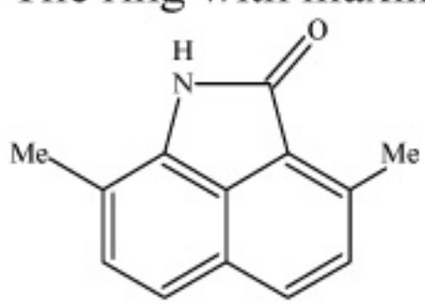
(C)



(D)

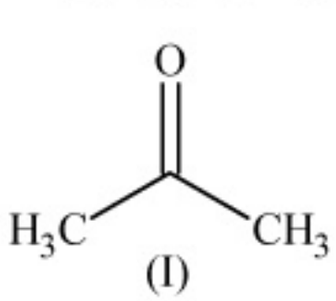


Ans. (B) The ring with maximum electron density will be substituted by an electrophile

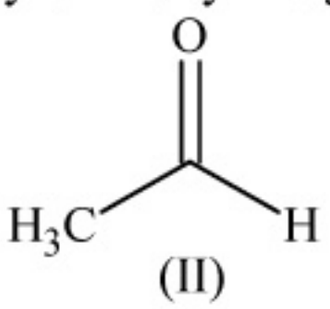


The ring attached with – NH- will have rich electron density due to resonance. As ortho position is blocked, the electrophile attacks the para position

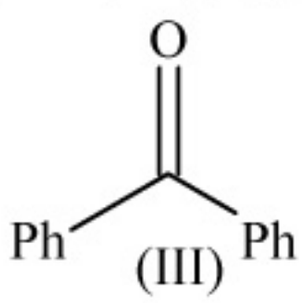
6. The order of reactivity of Phenyl Magnesium Bromide with the following compounds is



(I)



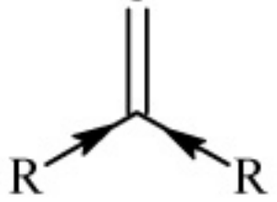
(II)

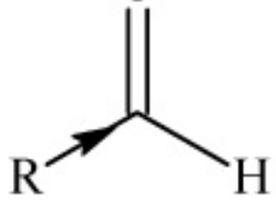


(III)

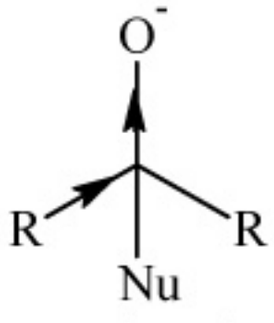
(A) (II) > (III) > (I)
(B) (I) > (III) > (II)
(C) (II) > (I) > (III)
(D) All react with the same rate

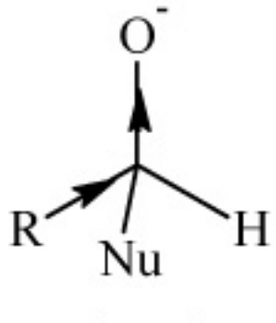
Ans. (C) Nucleophile attacks the most electrophilic site first. Among aldehyde and a ketone, aldehydes are more electrophilic as in ketones the δ+ charge an carbonyl carbon is decreased by +I effect of both alkyl groups





More over in the tetrahedral intermediate aldehydes have less steric repulsion than ketone and aldehydes increases the –ve charge on oxygen less in comparison to ketones





Based on the above the order of reactivity is (II) > (I) > (III)

7. $(NH_4)_2Cr_2O_7$ on heating gives a gas which is also given by
(A) Heating NH_4NO_2 (B) Heating NH_4NO_3
(C) $Mg_3N_2 + H_2O$ (D) Na(comp.) + H_2O_2

Ans. (A) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 \uparrow + Cr_2O_3 + 4H_2O$

$NH_4NO_2 \xrightarrow{\Delta} N_2 \uparrow + 2H_2O$

8. $Zn \mid Zn^{2+}(a = 0.1M) \parallel Fe^{2+}(a = 0.01M) \mid Fe$. The emf of the above cell is 0.2905 V. Equilibrium constant for the cell reaction is
(A) $10^{0.32/0.0591}$ (B) $10^{0.32/0.0295}$
(C) $10^{0.26/0.0295}$ (D) $e^{0.32/0.295}$

Ans. (B) $E = E° - \frac{0.0591}{n}\log Kc$

$0.2905 = E° - \frac{0.0591}{2}\log\frac{0.1}{0.01}$

$E° = 0.2905 + 0.0295 = 0.32$

$E° = \frac{0.0591}{n}\log K$

$0.32 = \frac{0.0591}{2}\log K$

$K = 10^{0.32/0.0295}$

9*. HX is a weak acid ($K_a = 10^{-5}$). It forms a salt NaX (0.1M) on reacting with caustic soda. The degree of hydrolysis of NaX is
(A) 0.01% (B) 0.0001%
(C) 0.1% (D) 0.5%

Ans. (A) $X^- + H_2O \longrightarrow HX + OH^-$

$K_h = \frac{K_w}{K_a} = 10^{-9}$

$K_h = C\,\alpha^2 = 10^{-9}$

$\alpha^2 = 10^{-8} \Rightarrow \alpha = 10^{-4}$

% degree of dissociation = $10^{-4} \times 100 = 0.01\%$

10. Spontaneous adsorption of a gas on solid surface is an exothermic process because
(A) ΔH increases for system (B) ΔS increases for gas
(C) ΔS decreases for gas (D) ΔG increases for gas

Ans. (C)For spontaneous absorption ΔG is negative as well as the degree of randomness of gas molecules decreases thereby ΔS also negative. That's why $T.\Delta S$ is –ve

$\therefore \Delta H = \Delta G + T.\Delta S$

Thereby ΔH is –ve

11*. For a monoatomic gas kinetic energy = E. The relation with rms velocity is
(A) $u = \left(\frac{2E}{m}\right)^{1/2}$ (B) $u = \left(\frac{3E}{2m}\right)^{1/2}$
(C) $u = \left(\frac{E}{2m}\right)^{1/2}$ (D) $u = \left(\frac{E}{3m}\right)^{1/2}$

Ans. (A) Since,

$P = \frac{1}{3}\times\frac{m}{v}\times c^2 \quad (c = \text{rms velocity}) \Rightarrow \sqrt{\frac{3PV}{m}} = c$

For 1molecule $PV = kT \Rightarrow c = \sqrt{\frac{3kT}{m}}$

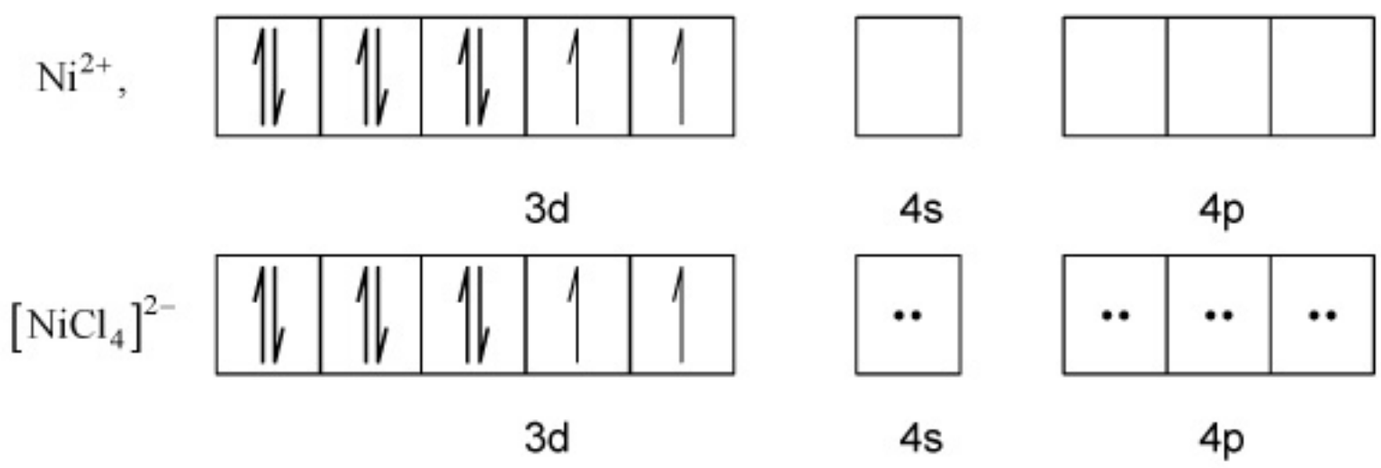
$\therefore KE = \frac{3}{2}kT \Rightarrow 2KE = 3kT \Rightarrow c = \sqrt{\frac{2KE}{m}}$

12*. The pair of compounds having metals in their highest oxidation state is
(A) MnO_2, $FeCl_3$
(B) $[MnO_4]^-$, CrO_2Cl_2
(C) $[Fe(CN)_6]^{3-}$, $[Co(CN)_3]$
(D) $[NiCl_4]^{2-}$, $[CoCl_4]^-$

Ans. (B)$[MnO_4]^-$, Mn = +7
CrO_2Cl_2, Cr = +6

13. The compound having tetrahedral geometry is
(A) $[Ni(CN)_4]^{2-}$
(B) $[Pd(CN)_4]^{2-}$
(C) $[PdCl_4]^{2-}$
(D) $[NiCl_4]^{2-}$

Ans. (D)



Hybridisation of $[NiCl_4]^{2-} = sp^3$
Shape of $[NiCl_4]^{2-}$ = Tetrahedral

14. Spin only magnetic moment of the compound $Hg[Co(SCN)_4]$ is
(A) $\sqrt{3}$
(B) $\sqrt{15}$
(C) $\sqrt{24}$
(D) $\sqrt{8}$

Ans. (B) $Hg[Co(SCN)_4]$

n=3
$\mu_s = \sqrt{n(n+2)} = \sqrt{3\times5} = \sqrt{15}$

15. A sodium salt of an unknown anion when treated with $MgCl_2$ gives white precipitate only on boiling. The anion is
(A) SO_4^{2-}
(B) HCO_3^-
(C) CO_3^{2-}
(D) NO_3^-

Ans. (B) $MgCl_2 + 2NaHCO_3 \longrightarrow \underset{\text{soluble}}{Mg(HCO_3)_2} + 2NaCl$

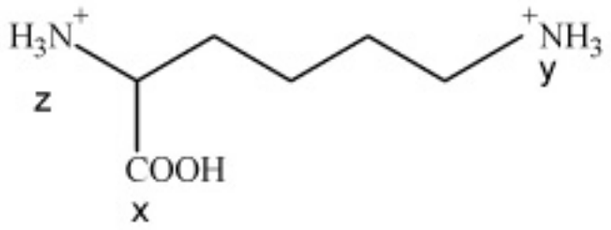
$Mg(HCO_3)_2 \xrightarrow{\Delta} \underset{\text{white}}{MgCO_3\downarrow} + H_2O + CO_2$

16*. Which Hydrogen like species will have same radius as that of Bohr orbit of Hydrogen atom?
(A) n=2, Li^{+2}
(B) n=2, Be^{3+}
(C) n=2, He^+
(D) n=3, Li^{2+}

Ans. (B) $r = \frac{n^2 r_{1(H^+)}}{z}$

$\because r = r_{(H^+)} \quad \therefore n^2 = z \quad \therefore n = 2, z = 4$

so $r_{2(Be^{+3})} = r_{1(H^+)}$

17. 

Arrange in order of increasing acidic strength

(A) X > Z > Y (B) Z < X > Y

(C) X > Y > Z (D) Z > X > Y

Ans. (A) pKa value of carboxylic group is less than pKa of $-NH_3(y)$ in amino acid and $-NH_3(z)$ will have comparatively less pKa than $-\overset{+}{N}H_3(z)$ due to –I effect of carboxylate group.

18. 0.004 M Na_2SO_4 is isotonic with 0.01 M Glucose. Degree of dissociation of Na_2SO_4 is

(A) 75% (B) 50%

(C) 25% (D) 85%

Ans. (A) $\pi_{Na_2SO_4} = \pi_{Glucose} = 0.01 \times RT$

or $0.01\ RT = i \times 0.004\ RT$

$i = 2.5$

$Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{-2}$

$1-\alpha \qquad 2\alpha \quad \alpha$

$i = 1+2\alpha = 2.5$

$\alpha = 0.75$

or 75% dissociation

19. $\Delta H_{vap.} = 30$ KJ/mole and $\Delta S_{vap.} = 75\ Jmol^{-1}K^{-1}$. Find temperature of vapour, at one atmosphere

(A) 400K (B) 350 K

(C) 298 K (D) 250 K

Ans. (A) $\Delta G = \Delta H - T\Delta S$

At equilibrium $\Delta G = 0$

$$T = \frac{\Delta H}{\Delta S} = \frac{30 \times 10^3}{75} = 400\ K$$

20. 2 mol of an ideal gas expanded isothermally & reversibly from 1 litre to 10 litres at 300 K. What is the enthalpy change?

(A) 4.98 KJ (B) 11.47 KJ

(C) -11.47 KJ (D) 0 KJ

Ans. (D) $H = E + PV$

and $\Delta H = \Delta E + \Delta(PV)$

or $\Delta H = \Delta E + nR\Delta T$

$\Delta T = 0$

$\Delta E = 0$

$\therefore \Delta H = 0$

21*. (A) follows first order reaction. (A) $\longrightarrow$ product

Concentration of A, changes from 0.1 M to 0.025 M in 40 minutes. Find the rate of reaction of A when concentration of A is 0.01 M

(A) 3.47×10^{-4} M min^{-1} (B) 3.47×10^{-5} M min^{-1}

(C) 1.73×10^{-4} M min^{-1} (D) 1.73×10^{-5} M min^{-1}

Ans. (A)Concentration changes from 0.01 to 0.025 M in 40 minutes $\Rightarrow 2t_{1/2} = 40\,\text{min}$

$t_{1/2} = 20\,\text{min}$

$$r = K[A] = \frac{0.693}{20} \times 0.01 = 3.47 \times 10^{-4}$$

22*. 2-hexyne gives trans-2-hexene on treatment with

(A) Li / NH_3 (B) $Pd/ BaSO_4$

(C) $LiAlH_4$ (D) Pt/H_2

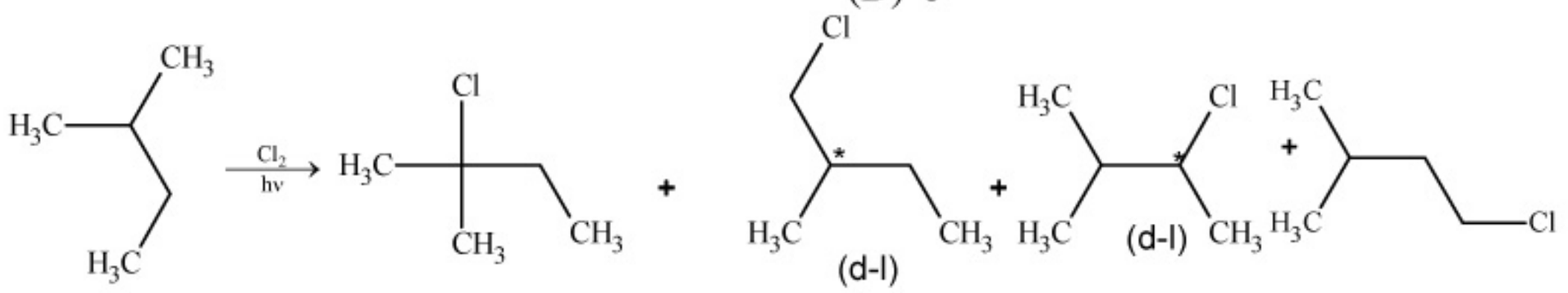
Ans. (A) Li/NH_3 brings about trans addition of H_2

23*. How many chiral compounds are possible on mono chlorination of 2-methyl butane?

(A) 2 (B) 4

(C) 6 (D) 8

Ans. (B)



24. Which of the following pairs give positive Tollen's test?

(A) Glucose, sucrose (B) Glucose, fructose

(C) Hexanal, Acetophenone (D) Fructose, sucrose

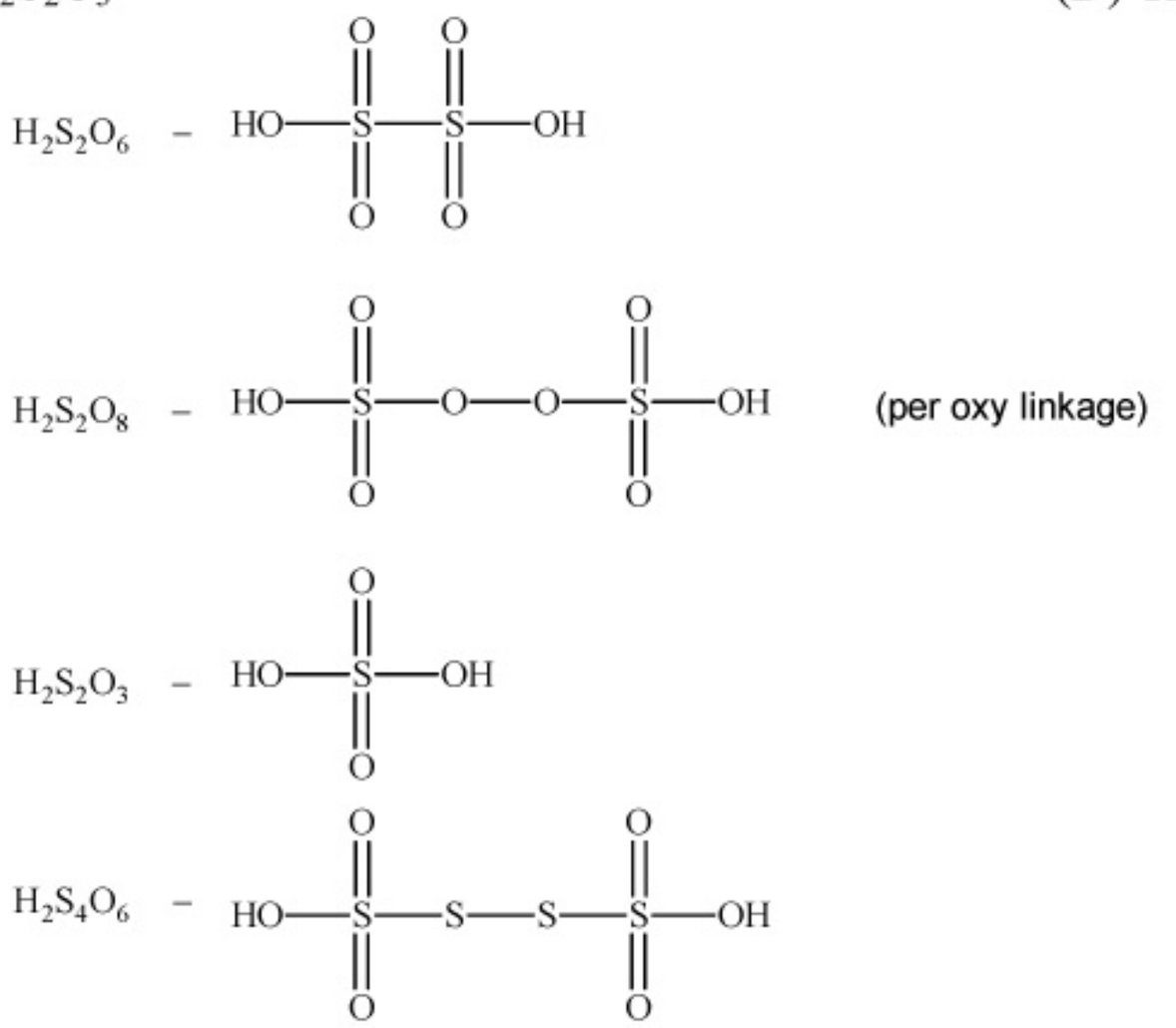
Ans. (B) Aldehydes and α- hydroxy ketones give positive Tollen's test. Glucose has an aldehydic group and fructose is an α hydroxy ketone

25. Which of the following has – O – O – linkage

(A) $H_2S_2O_6$ (B) $H_2S_2O_8$

(C) $H_2S_2O_3$ (D) $H_2S_4O_6$

Ans. (B)



26. When I^- is oxidised by MnO_4^- in alkaline medium, I^- converts into

(A) IO_3^- (B) I_2

(C) IO_4^- (D) IO^-

Ans. (A) $2KMnO_4 + 2KOH \longrightarrow 2K_2MnO_4 + H_2O + O$

$$2K_2MnO_4 + 2H_2O \longrightarrow 2MnO_2 + 4KOH + 2O$$

$$2KMnO_4 + H_2O \xrightarrow{\text{alkaline}} 2MnO_2 + 2KOH + 3[O]$$

$$KI + [O] \longrightarrow KIO_3$$

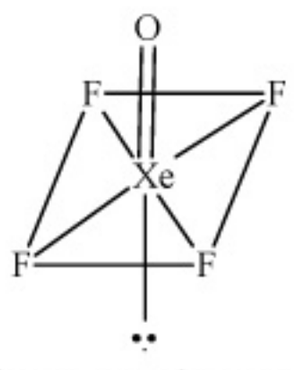
$$2KMnO_4 + KI + H_2O \longrightarrow 2KOH + 2MnO_2 + KIO_3$$

27*. Number of lone pair(s) in $XeOF_4$ is/are

(A) 0 (B) 1

(C) 2 (D) 3

Ans. (B) Structure of $XeOF_4$



Number of lone pairs on the central atom is 1.

28*. According to MO Theory,

(A) O_2^+ is paramagnetic and bond order greater than O_2

(B) O_2^+ is paramagnetic and bond order less than O_2

(C) O_2^+ is diamagnetic and bond order is less than O_2

(D) O_2^+ is diamagnetic and bond order is more than O_2

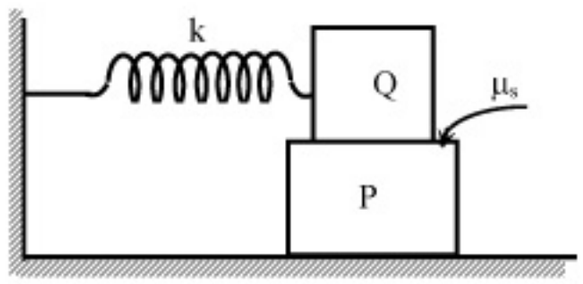
Ans. (A) O_2^+ B.O. $= \frac{1}{2}[\text{no. of bonding} - \text{no. of antibonding electrons}] = \frac{1}{2}[10-5] = 2.5$

O_2 B.O. $= \frac{1}{2}[10-6]$

$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2pz^2 \begin{bmatrix} \pi 2px^2 \\ \pi 2py^2 \end{bmatrix} \begin{bmatrix} \pi^* 2px^1 \\ \pi^* 2py^1 \end{bmatrix}$$

One unpaired e^- hence paramagnetic

29*. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks.



(A) kA/2 (B) kA

(C) $\mu_s mg$ (D) zero

Ans. (A) $a_{max} = \frac{k}{2m}A$, hence $f_{max} = m\, a_{max} = \frac{kA}{2}$

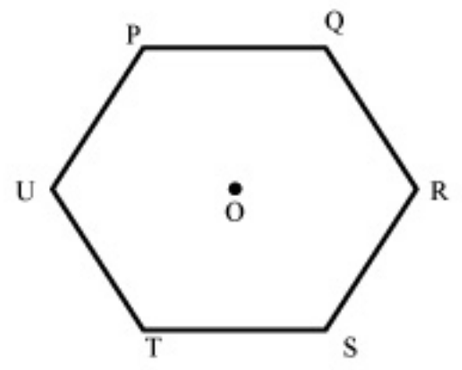
30. A beam of white light is incident on glass air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are

(A) Violet, Indigo, Blue (B) All coolers except green

(C) Yellow, Orange, Red (D) White light

Ans. (C) Condition for light to transmit is $\sin C < 1/\mu$, $\mu_v > \mu_g > \mu_r$

31. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on PQRSTU corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R

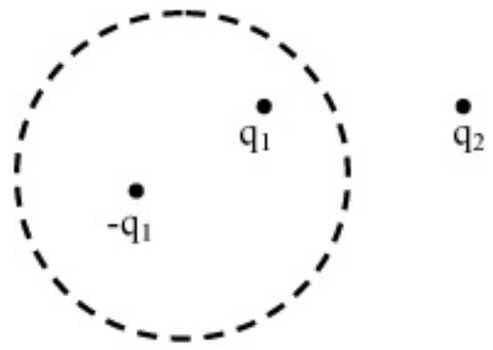


(A) +, +, +, -, -, - (B) -, +, +, +, -, -

(C) -, +, +, -, +, - (D) +, -, +, -, +, -

Ans. (C)

32. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be



(A) due to q_1 and q_2 only
(B) due to q_2 only
(C) zero
(D) due to all

Ans. (D)

33*. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be

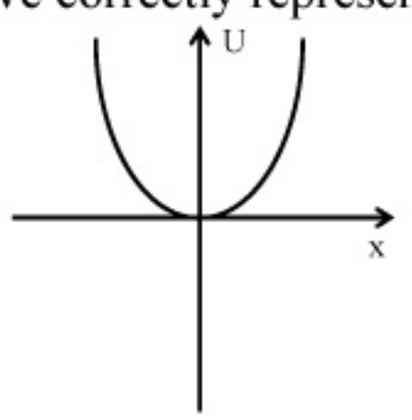
(A) 2K
(B) K/2
(C) K
(D) K/4

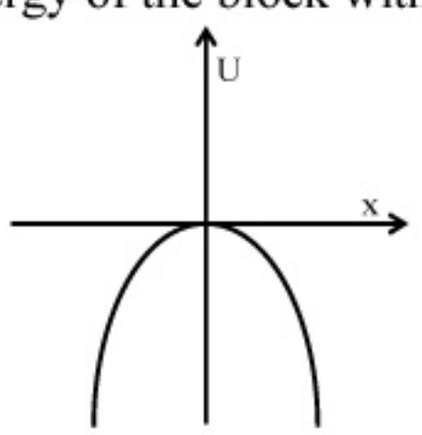
Ans. (B) $I\omega_0 = 2I\omega' \Rightarrow \omega' = \omega_0 / 2$

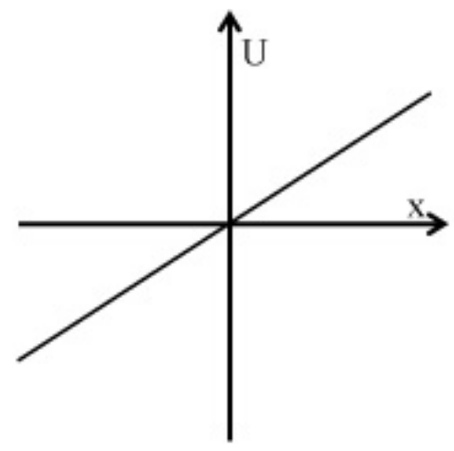
$$K = \frac{1}{2} I\omega_0^2$$

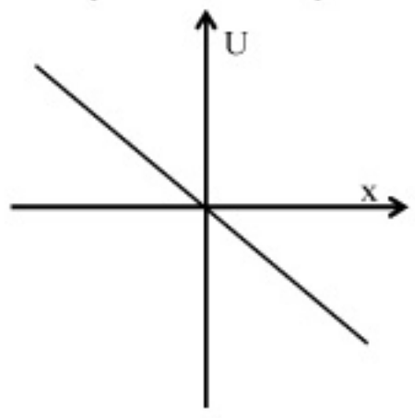
$$K' = \frac{1}{2} 2I\left(\frac{\omega_0}{2}\right)^2 = \frac{K}{2}$$

34*. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x

(A)


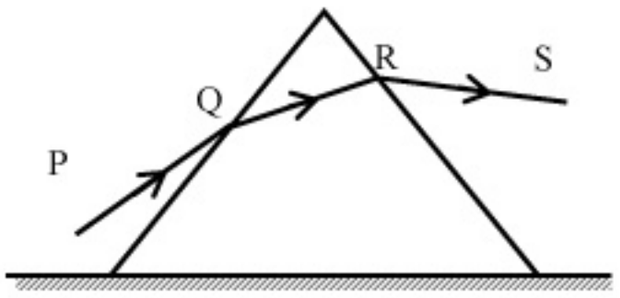
(B)


(C)


(D)


Ans. (B) $U = -\int_0^x kx\,dx = -\frac{kx^2}{2}$

35. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation



(A) PQ is horizontal
(B) QR is horizontal
(C) RS is horizontal
(D) Any one will be horizontal.

Ans. (B) For minimum deviation, i = e

36*. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal.

(A) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$
(B) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$
(C) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$
(D) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

Ans. (B) $\ell_1 = \frac{3}{4}\frac{v_1}{f_1}$, $\ell_2 = \frac{v_2}{f_2}$

$$\frac{3v_1}{4\ell_1} = \frac{v_2}{\ell_2}$$

$$\ell_2 = \frac{4\ell_1 v_2}{3v_1} = \frac{4\ell_1}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

37*. Three discs A, B and C having radii 2, 4, and 6 cm respectively are coated with carbon black. Wavelength for maximum intensity for the three discs are 300, 400 and 500 nm respectively. If Q_A, Q_B and Q_C are power emitted by A, B and C respectively, then

(A) Q_A will be maximum (B) Q_B will be maximum
(C) Q_C will be maximum (D) $Q_A = Q_B = Q_C$

Ans. (B) $\lambda_m T$ = constant

$$T_1 : T_2 : T_3 :: \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$$

$$Q = \sigma\varepsilon A T^4$$

$$Q_A : Q_B : Q_C :: \frac{2^2}{3^4} : \frac{4^2}{4^4} : \frac{6^2}{5^4}$$

Q_B will be maximum.

38. Monochromatic light of wavelength 400 nm and 560 nm are incident simultaneously and normally on double slits apparatus whose slits separation is 0.1 mm and screen distance is 1m. Distance between areas of total darkness will be

(A) 4 mm (B) 5.6 mm
(C) 14 mm (D) 28 mm

Ans. (D) $(2n + 1)\lambda_1 = (2m + 1)\lambda_2$

$$\frac{2n+1}{2m+1} = \frac{560}{400} = \frac{7}{5}$$

$10\,n = 14m + 2$

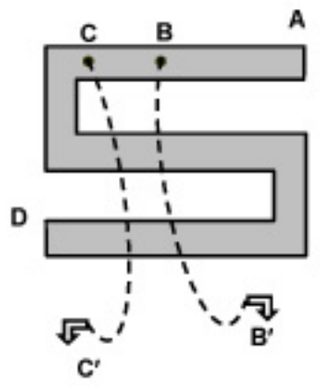
By inspection, for $m = 2$; $n = 3$

$m = 7$; $n = 10$

$$\therefore\ \Delta s = \frac{\lambda_1 D}{2d}[(2n_2 + 1) - (2n_1 + 1)] = 28\,\text{mm}.$$

39. Shown in figure is a Post Office box. In order to calculate the value of external resistance, it should be connected between

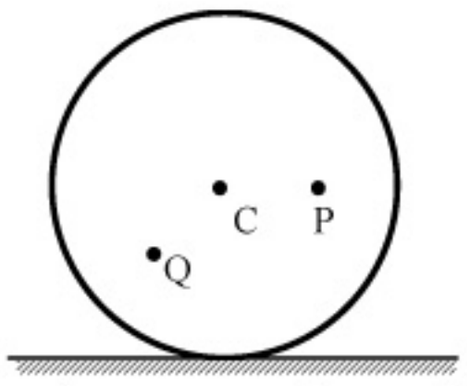
(A) B′ and C′ (B) A and D
(C) C and D (D) B and D



Ans. (B)

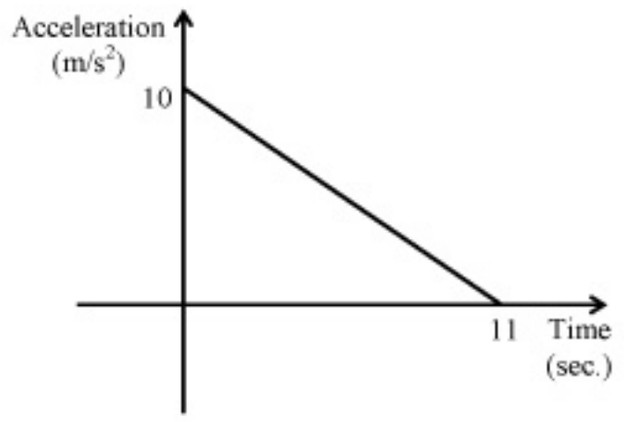
40*. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is

(A) $v_Q > v_C > v_P$ (B) $v_P > v_C > v_Q$
(C) $v_P = v_C$, $v_Q = v_C/2$ (D) $v_P < v_C > v_Q$



Ans. (B) About instantaneous axis of rotation i.e. point of contact $v = r\omega$, r_P is maximum $\therefore$ v_P is maximum.

41*. A body starts from rest at time t = 0, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be



(A) 110 m/s
(B) 55 m/s
(C) 650 m/s
(D) 550 m/s

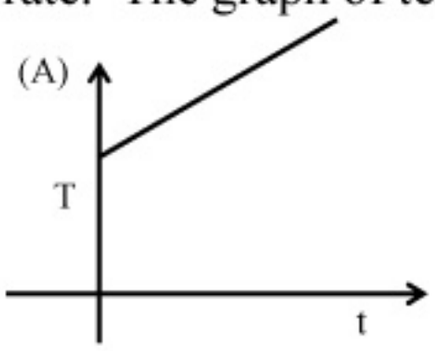
Ans. (B) Maximum velocity = area under the graph = 55 m/s.

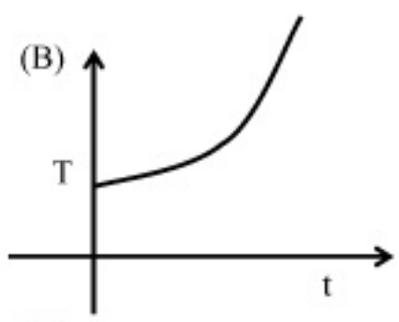
42*. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m/s, velocity of sound in air = 300 m/s)

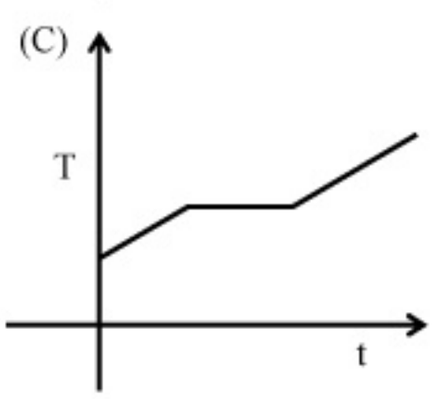
(A) 3000 Hz
(B) 120 Hz
(C) 600Hz
(D) 6000 Hz

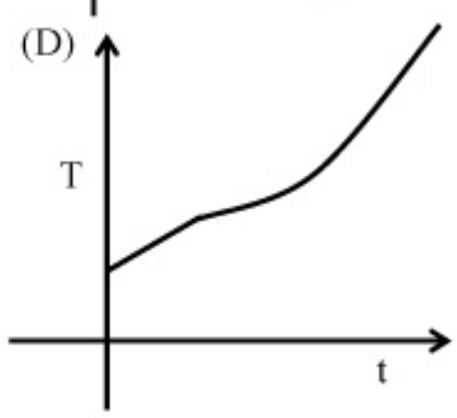
Ans. (C)

43*. If liquefied oxygen at 1 atmospheric pressure is heated from 50 k to 300 k by supplying heat at constant rate. The graph of temperature vs time will be



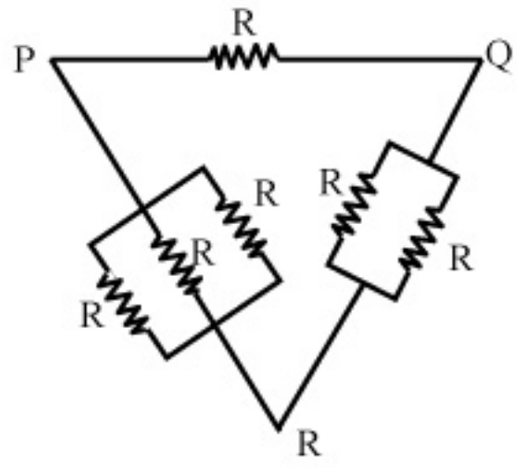






Ans. (C)

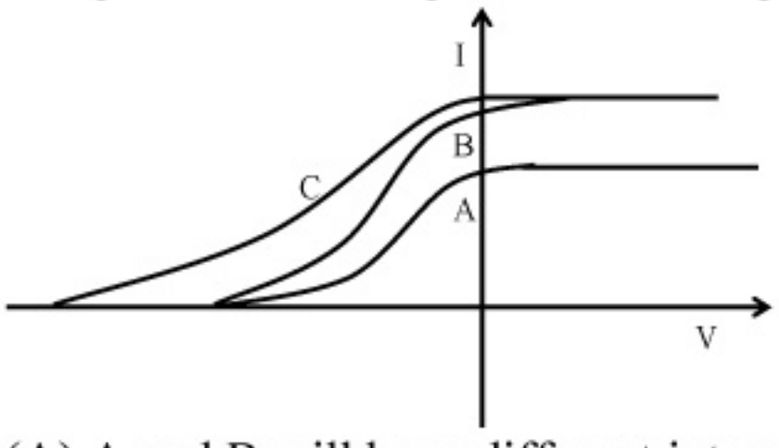
44. Six identical resistors are connected as shown in the figure. The equivalent resistance will be



(A) Maximum between P and R.
(B) Maximum between Q and R.
(C) Maximum between P and Q.
(D) all are equal.

Ans. (C)

45. In a photoelectric experiment anode potential is plotted against plate current



(A) A and B will have different intensities while B and C will have different frequencies.
(B) B and C will have different intensities while A and C will have different frequencies.
(C) A and B will have different intensities while A and C will have equal frequencies.
(D) A and B will have equal intensities while B and C will have different frequencies.

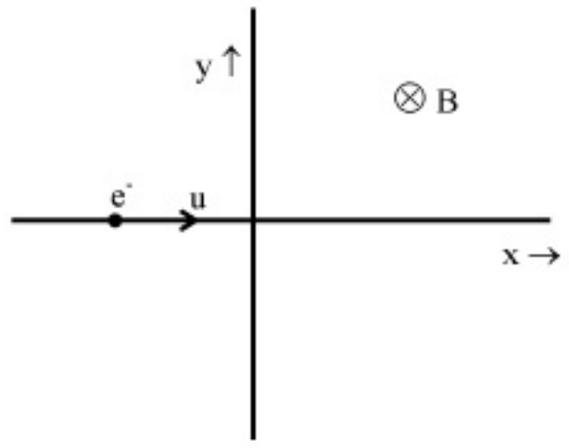
Ans. (A)

46*. Pressure depends on distance as, $P = \frac{\alpha}{\beta}\exp\left(-\frac{\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzman's constant and θ is temperature. The dimension of β are

(A) $M^0L^0T^0$ (B) $M^{-1}L^{-1}T^{-1}$
(C) $M^0L^2T^0$ (D) $M^{-1}L^1T^2$

Ans. (C) $\frac{\alpha z}{k\theta}$ should be dimensionless, hence $\alpha = MLT^{-2}$
$\alpha / \beta = ML^{-1}T^{-2} = P$, hence $\beta = M^0L^2T^0$

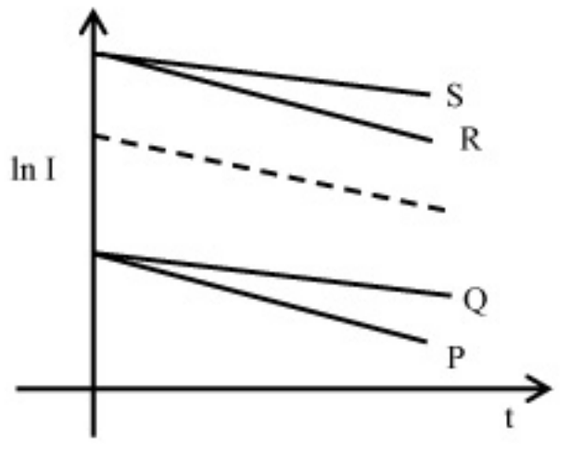
47. An electron traveling with a speed u along the positive x-axis enters into a region of magnetic field where $B = -B_0\ \hat{k}$ $(x > 0)$. It comes out of the region with speed v then



(A) $v = u$ at $y > 0$ (B) $v = u$ at $y < 0$
(C) $v > u$ at $y > 0$ (D) $v > u$ at $y < 0$

Ans. (B) Charged particle will move in a circular path with constant speed inside the magnetic field.

48. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln I with respect to time. If the resistance is changed to 2x, the new graph will be



(A) P (B) Q
(C) R (D) S

Ans. (B) $I = I_0\, e^{-t/xC} \Rightarrow \ln I = \ln I_0 - t/xC$
I_0 is inversely proportional to x.

49. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later it's activity becomes 3000dps. What was its initial activity

(A) 20000 dps (B) 24000 dps
(C) 12000 dps (D) 6000 dps

Ans. (B) $A_1 = \lambda N_1 = \lambda N_0\, e^{-\lambda t1}$, $A_2 = \lambda N_2 = \lambda N_0\, e^{-\lambda t2}$ and $A_0 = \lambda N_0 = 24000$ dps.

50*. Two identical rods are connected between two containers one of them is at 100^0C and another is at 0^0C. If rods are connected in parallel then the rate of melting of ice q_1 gm/sec. If they are connected in series then rate is q_2. Then the ratio q_2 / q_1 is

(A) 2 (B) 4
(C) ½ (D) ¼

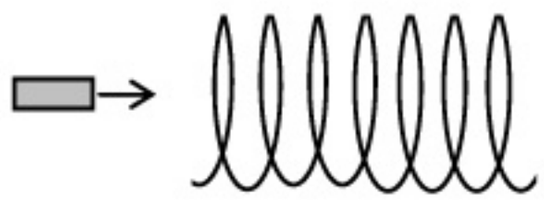
Ans. (D) $q_1 = \frac{CT}{R_T/2}$, $q_2 = \frac{CT}{2R_T}$ and $q_2 / q_1 = 1/4$

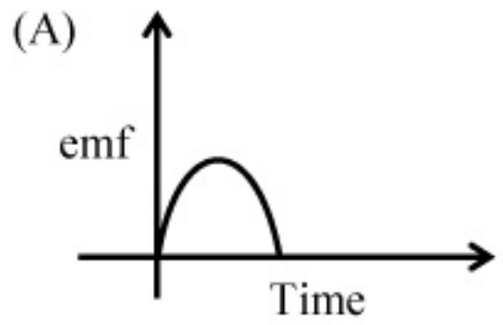
51*. A particle starts sliding down a frictionless inclined plane. If S_n is the distance traveled by it from time t = n- 1 sec to t = n sec, the ratio S_n / S_{n+1} is

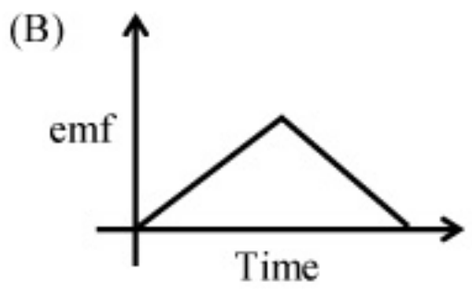
(A) $\frac{2n-1}{2n+1}$ (B) $\frac{2n+1}{2n}$
(C) $\frac{2n}{2n+1}$ (D) $\frac{2n+1}{2n-1}$

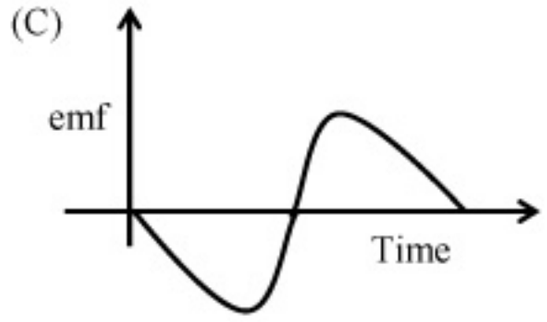
Ans. (A) $S_n = \frac{a}{2}(2n-1)$, $S_{n+1} = \frac{a}{2}(2n+2-1)$

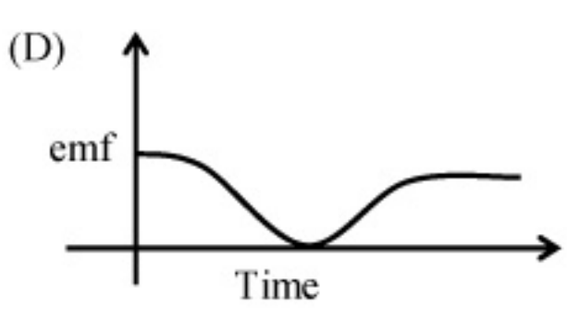
52. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced with time











Ans. (C)

53. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is

(A) 6 cm (B) 4 cm
(C) 12 cm (D) 9 cm

Ans. (A) Object is at centre of curvature, hence image will also be at centre of curvature.

54. A proton has kinetic energy E = 100 keV which is equal to that of a photon. The wavelength of photon is λ_2 and that of proton is λ_1. The ratio of λ_1 / λ_2 is proportional to

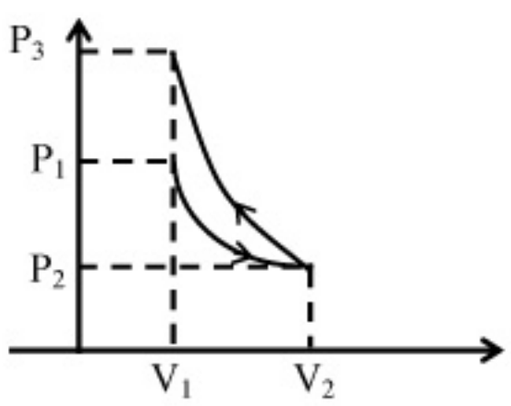
(A) E^2 (B) $E^{1/2}$
(C) E^{-1} (D) $E^{-1/2}$

Ans. (B) $\lambda_{proton} = \dfrac{h}{\sqrt{2mE}}$, $\lambda_{photon} = \dfrac{hc}{E}$.

55*. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process which of the following is true

(A) $W > 0$; $P_3 > P_1$ (B) $W < 0$; $P_3 > P_1$
(C) $W > 0$; $P_3 < P_1$ (D) $W < 0$; $P_3 < P_1$

Ans. (B)



56*. A wire of length $\ell = 6 \pm 0.06$ cm and radius $r = 0.5 \pm 0.005$ cm and mass $m = 0.3 \pm 0.003$ gm. Maximum percentage error in density is

(A) 4 (B) 2
(C) 1 (D) 6.8

Ans. (A) $\rho = \dfrac{m}{\ell \pi r^2}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell} = \frac{0.003}{0.3} + \frac{2 \times 0.005}{0.5} + \frac{0.06}{6} = \frac{4}{100} = 4\ \%$$

57*. The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the angles of the triangle are in the ratio
(A) 1 : 3 : 5 (B) 2 : 3 : 4
(C) 3 : 2 : 1 (D) 1 : 2 : 3

Ans. (D) Let $a = x, b = \sqrt{3}\,x, c = 2x$
$c^2 = a^2 + b^2 \Rightarrow \angle C = 90^0$
$\tan A = \frac{1}{\sqrt{3}} \Rightarrow \angle A = 30^0.$
$\Rightarrow A : B : C = 1 : 2 : 3.$

58*. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is
(A) 2 sq. units (B) 4 sq. units
(C) 6 sq. units (D) 8 sq. units

Ans. (A) $x^2 - y^2 + 2y - 1 = 0$
$\Rightarrow$ Equations of lines are $y = x + 1, y = -x + 1$
$\Rightarrow$ angle bisectors are $y = 1$ and $x = 0$
$\Rightarrow$ area of triangle $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units

59. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is
(A) 4/25 (B) 4/35
(C) 4/33 (D) 4/1155

Ans. (D) Numbers between 1 and 100 which are divisible by both 2 and 3 are 16.
Hence the probability is $\frac{^{16}C_3}{^{100}C_3} = \frac{4}{1155}$.

60. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is
(A) $1/\sqrt{3}$ (B) 1/2
(C) 1 (D) 1/3

Ans. (A) Points of intersection of $y = ax^2$ and $x = ay^2$ are (0, 0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$.
Hence $\int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx = 1 \Rightarrow a = \frac{1}{\sqrt{3}}$ (as $a > 0$).

61*. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to
(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$
(C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left(\frac{5\pi}{6}, \pi\right]$

Ans. (B) $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$
$\cos\phi = \frac{1}{3} \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2} \Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$.

62*. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is

(A) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (B) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

(C) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (D) $\frac{x^2}{4}+\frac{y^2}{2}=1$

Ans. (A) Equation of tangent at any point 'θ' is $\frac{x}{\sqrt{2}}\cos\theta + y\sin\theta = 1$ and the midpoint of its intercept between the axes is $\left(\frac{\sqrt{2}}{2}\sec\theta,\ \frac{1}{2}\operatorname{cosec}\theta\right) \Rightarrow$ locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$.

63. If $f(x)$ is differentiable and $\int_0^{t^2} x\,f(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) –5/2
(C) 1 (D) 5/2

Ans. (A) Differentiating both sides, we get

$t^2\,f(t^2).2t = \frac{5t^4\cdot 2}{5} \Rightarrow f(t^2) = t \Rightarrow f\left(\frac{4}{25}\right) = \pm\frac{2}{5}$.

64*. The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ is

(A) 1/2 (B) 1
(C) 0 (D) –1/2

Ans. (D) $\frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}} \Rightarrow x^2+2x+2 = x^2+1 \Rightarrow x = -\frac{1}{2}$.

65. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(A) f (x) is a strictly increasing function (B) f(x) has a local maxima
(C) f(x) is a strictly decreasing function (D) f(x) is bounded

Ans. (A) $f'(x) = 3x^2 + 2bx + c$

$D = 4b^2 - 12c = 4(b^2 - c) - 8c \Rightarrow D < 0 \Rightarrow f'(x) > 0\ \forall\ x \in (-\infty, \infty) \Rightarrow f(x)$ is an increasing function.

66*. If $\omega\ (\neq 1)$ be a cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive value of n is

(A) 2 (B) 3
(C) 5 (D) 6

Ans. (B) $(1+\omega^2)^n = (1+\omega^4)^n \Rightarrow (-\omega)^n = (-\omega^2)^n \Rightarrow (\omega)^n = 1 \Rightarrow n = 3$.

67. If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1] is

(A) -2 (B) -1
(C) 0 (D) 1/2

Ans. (D) For function to satisfy the condition of Rolle's theorem, it should be continuous in [0, 1]

$\Rightarrow \lim_{x\to 0^+} f(x) = f(0) \Rightarrow \lim_{x\to 0^+} \frac{\log x}{x^{-\alpha}} = 0 \Rightarrow \lim_{x\to 0^+} \frac{1/x}{-\alpha x^{-\alpha-1}} = 0 \Rightarrow \alpha > 0$.

Also $\forall\ \alpha > 0$, f(x) is differentiable in (0, 1) and $f(1) = 0 = f(0)$.

68*. For all 'x', $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is

(A) $a < -5$ (B) $-5 < a < 2$

(C) $a > 5$ (D) $2 < a < 5$

Ans. (B) $D < 0 \Rightarrow 4a^2 - 4(10 - 3a) < 0 \Rightarrow 4a^2 + 12a - 40 < 0 \Rightarrow -5 < a < 2$.

69*. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(A) $\pi/6$ (B) $\pi/4$

(C) $\pi/3$ (D) $\pi/2$

Ans. (C) Equation of tangent is $y = mx + \frac{1}{m}$.

Since it passes through (1, 4)

$\therefore m^2 - 4m + 1 = 0 \Rightarrow m_1 + m_2 = 4, m_1 m_2 = 1 \Rightarrow |m_1 - m_2| = 2\sqrt{3}$

$\therefore \tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$.

70*. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) + q^2 = 0$ (D) $p^3 + q(3p + 1) + q^2 = 0$

Ans. (A) Let the roots be α, α^2

$\Rightarrow \alpha^2 + \alpha = -p, \alpha^3 = q \Rightarrow \alpha(\alpha + 1) = -p \Rightarrow \alpha^3(\alpha^3 + 1 + 3(\alpha^2 + \alpha)) = -p^3 \Rightarrow p^3 - q(3p - 1) + q^2 = 0$.

71. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is

(A) $\frac{\pi}{2} + 1$ (B) $\frac{\pi}{2} - 1$

(C) -1 (D) 1

Ans. (B) Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}dx$. Put $x = \cos\theta \Rightarrow dx = -\sin\theta\, d\theta$ then $I = \int_0^{\pi/2} (1 - \cos\theta)d\theta = \frac{\pi}{2} - 1$.

72. If $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is

(A) $\hat{i} - \hat{j} + \hat{k}$ (B) $2\hat{j} - \hat{k}$

(C) $\hat{i}$ (D) $2\hat{i}$

Ans. (C) $\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}$

$(\hat{i} + \hat{j} + \hat{k}) \times (\hat{j} - \hat{k}) = (\hat{i} + \hat{j} + \hat{k}) - 3\vec{b} \Rightarrow \vec{b} = \hat{i}$.

73*. If ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$, then $k \in$

(A) $(-\infty, -2]$ (B) $[2, \infty)$

(C) $\left[-\sqrt{3}, \sqrt{3}\right]$ (D) $\left(\sqrt{3}, 2\right]$

Ans. (D) ${}^{n-1}C_r = (k^2 - 3)\frac{n}{r+1}\,{}^{n-1}C_r \Rightarrow k^2 - 3 = \frac{r+1}{n}$

$\therefore 0 < k^2 - 3 \le 1$ or $3 < k^2 \le 4$.

74. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

(A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $[0, \pi]$

Ans. (B) $g(f(x)) = (\sin x + \cos x)^2 - 1 = \sin 2x$ which is invertible in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$.

75. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$, $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals

(A) 1/3 (B) 2/3

(C) -1/3 (D) 1

Ans. (A) $\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

$\ln(y+1) = -\ln(2+\sin x) + \ln c \Rightarrow y + 1 = \frac{c}{2+\sin x}$. Putting $x = 0$ and $y = 1$, we get $c = 4$

$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{1}{3}$.

76. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is

(A) 3/2 (B) 9/2

(C) –2/9 (D) –3/2

Ans. (B) General points on the lines are $(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$ and $(\mu + 3, 2\mu + k, \mu)$
Equating the corresponding coordinates, we get $k = 9/2$.

77. Given $2x - y - 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the value of λ such that the given system of equation has NO solution, is

(A) 3 (B) 1

(C) 0 (D) – 3

Ans. (D) As $\Delta_z \neq 0$, for no solution $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -3$.

78*. If the line $2x + \sqrt{6}\,y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is

(A) $(-2, \sqrt{6})$ (B) $\left(-5, 2\sqrt{6}\right)$

(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (D) $\left(4, -\sqrt{6}\right)$

Ans. (D) Equation of tangent is $xx_1 - 2yy_1 = 4$
on comparing with $2x + \sqrt{6}\,y = 2$, we get $x_1 = 4$ and $y_1 = -\sqrt{6}$.

79. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is

(A) ± 1 (B) ± 2

(C) ± 3 (D) ± 5

Ans. (C) $|A|^3 = 125 \Rightarrow |A| = 5 \Rightarrow \alpha = \pm 3$.

80. The unit vector which is orthogonal to the vector $5\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

(A) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\frac{2\hat{i}-5\hat{j}}{\sqrt{29}}$

(C) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (D) $\frac{2\hat{i}-8\hat{j}+\hat{k}}{\sqrt{69}}$

Ans. (C) Let $\vec{a}=5\hat{i}+2\hat{j}+6\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-\hat{j}+\hat{k}$, then required unit vector will be along $\vec{a}\times(\vec{b}\times\vec{c})$.

$\vec{a}\times(\vec{b}\times\vec{c}) = 27\hat{j}-9\hat{k} \Rightarrow$ unit vector is $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$.

81. If f(x) is differentiable and strictly increasing function , then the value of $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is

(A) 1 (B) 0

(C) –1 (D) 2

Ans. (C) Using L'Hospital's rule

$\lim_{x\to 0}\frac{2xf'(x^2)-f'(x)}{f'(x)} = -1 \quad (\because\ f'(x) > 0\ \forall\ x)$

82*. An infinite G.P. has first term 'x' and sum '5', then x belongs to

(A) $x < -10$ (B) $-10 < x < 0$

(C) $0 < x < 10$ (D) $x > 10$

Ans. (C) The sum of an infinite G.P. $= \frac{x}{1-r} = 5$ (given)

$|r| < 1 \Rightarrow \left|1-\frac{x}{5}\right| < 1 \Rightarrow 0 < x < 10.$

83*. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

(A) $\sqrt{3}$ (B) $\sqrt{2}$

(C) 3 (D) 2

Ans. (C) Centre is (1, 3) and radius = 2

If r = radius of second circle then $r^2 = 2^2 + (3-1)^2 + (2-1)^2 \Rightarrow r = 3$.

84. If y is a function of x and $\log(x + y) - 2xy = 0$, then the value of $y'(0)$ is equal to

(A) 1 (B) –1

(C) 2 (D) 0

Ans. (A) At x = 0, y = 1

$\log(x + y) - 2xy = 0$

$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2y(x+y)-1}{1-2(x+y)x} \Rightarrow \left.\frac{dy}{dx}\right|_{(0,\,1)} = 1$.