SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2012 – PAPER II VERSION – B1

[MATHEMATICS]

1. Ans: $2^{35\times 35}$

Sol: n(A) = 5, n(B) = 7
No. of relations = $2^{n(A\times B)\times n(A\times B)}$
$= 2^{35\times 35}$.

2. Ans: $\phi(a) + \phi(b)$

Sol: $\phi(a + b) = b[a + b - a] + a[a + b - b]$

$$= \frac{b^2}{b-a} + \frac{a^2}{a-b}$$

$$= \frac{b^2}{b-a} - \frac{a^2}{b-a} = a + b$$

$= \phi(a) + \phi(b)$.

3. Ans: (1, 2]

Sol: Range $y = \dfrac{x^2+8}{x^2+4}$

$x = \dfrac{2\sqrt{2-y}}{\sqrt{y-1}}$ $\quad y - 1 > 0$

$2 - y \geq 0$.

4. Ans: $1000 \leq p \leq 1499$

Sol: n(A) = 1000, n(B) = 500
$n(A \cap B) \geq 1 \quad n(A \cup B) = p$
$p = n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 1000 + 500 - n(A \cap B)$
$\because n(A \cap B) \geq 1 \quad p \leq 1499$
$p \geq 1000$.

5. Ans: [–7, –3]

Sol: $-1 \leq \dfrac{x+5}{2} \leq 1$
$-2 \leq x + 5 \leq 2$
$-7 \leq x \leq -3$.

6. Ans: 2x + 1

Sol: f(x) = x + 1
g(x) = 2x
f(g(x)) = f(2x)
= 2x + 1.

7. Ans: 1

Sol: $\left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} + \dfrac{1}{z_4}\right| = 1$

$$\Rightarrow \left|\frac{1}{e^{i\theta_1}} + \frac{1}{e^{i\theta_2}} + \frac{1}{e^{i\theta_2}} + \frac{1}{e^{i\theta_3}} + \frac{1}{e^{i\theta_4}}\right| = 1$$

$$\Rightarrow \left|e^{-i\theta_1} + e^{-i\theta_2} + e^{-i\theta_2} + e^{-i\theta_3} + e^{-i\theta_4}\right| = 1$$

$$\Rightarrow \left|e^{-i(\theta_1+\theta_2+\theta_3+\theta_4)}\right| = 1$$

$\Rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$

$$\left|e^{i\theta_1} + e^{i\theta_2} + e^{i\theta_3} + e^{i\theta_4}\right| = \left|e^{i(\theta_1+\theta_2+\theta_3+\theta_4)}\right|$$

$= e^{i0} = 1$.

8. Ans: $-\dfrac{1}{2} + i\dfrac{3\sqrt{3}}{2}$

Sol: $z = \omega$
$8 + 10\omega + 7\omega^2 = 1 + 7 + 7\omega + 3\omega + 7\omega^2$
$= 7 + 7\omega + 7\omega^2 + 1 + 3\omega$
$= 7(1 + \omega + \omega^2) + 1 + 3\omega$
$= 0 + 1 + 3\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)$

$$= 1 - \frac{3}{2} + i\frac{3\sqrt{3}}{2} = \frac{-1}{2} + i\frac{3\sqrt{3}}{2}.$$

9. Ans: |z| = 1

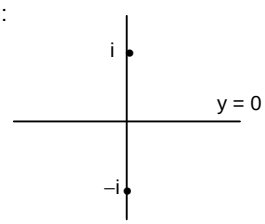
Sol: $\dfrac{\omega - \overline{\omega}z}{1-z} = r(\text{real})$

$\Rightarrow z = \dfrac{(r-x)-iy}{(r-x)+iy}$

$\Rightarrow |z| = 1$.

10. Ans: y = 0

Sol:



$$\left|\frac{1+iz}{z+i}\right| = \left|\frac{i(z-i)}{z-(-i)}\right| = 1$$

$$\Rightarrow \left|\frac{z-i}{z-(-i)}\right| = 1$$

$\Rightarrow$ perpendicular bisector between points i and –i.

$\Rightarrow y = 0$.

11. Ans: $i - i^{n+2}$

Sol: Put n = 1

$$\sum_{k=0}^{1}\left(i^k + i^{k+1}\right) = i^0 + i^1 + i^1 + i^2$$

$= 1 + i + i - 1$

$= 2i$

In option D, put n = 1, $i - i^3$

$= i - (-i)$

$= 2i.$

12. Ans: $|z_1 + z_2|$

Sol: $|z_1| = 1 \Rightarrow \left|\frac{1}{z_1}\right| = 1$

$|z_2| = 1 \Rightarrow \frac{1}{|z_2|} = 1$

$\left|\frac{1}{z_1} + \frac{1}{z_2}\right| \le \left|\frac{1}{z_1}\right| + \left|\frac{1}{z_2}\right|.$

13. Ans: $c < 0$

Sol: $\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}, a > 0$

$\alpha < -5$ and $\beta > 5 \Rightarrow \alpha\beta < 0$

but $a > 0$, $\therefore c < 0$.

14. Ans: $\frac{17}{8}$

Sol: Let $ax^2 + bx + c = 0$ be $x^2 + 2x + 1$

$\alpha = -1, \beta = -1$

Given expression becomes $\frac{17}{8}$.

15. Ans: $x^2 - (a + b)x + ab = 0$

Sol: $\alpha^2 + a\alpha + bc = 0$

$\alpha^2 + b\alpha + ca = 0$

$a\alpha - b\alpha + bc - ca = 0$

$\alpha(a - b) + c(b - a) = 0$

$\alpha = c$

From equation (1) $\alpha\beta = bc$

$\Rightarrow \beta = b$

From equation (2)

$\alpha\beta = ca$

$\Rightarrow \beta = a$

$\therefore$ required equation is

$x^2 - (a + b)x + ab = 0.$

16. Ans: $y^2 - 8y + 9 = 0.$

Sol: $y = x + \frac{1}{x}$

$$\left(\frac{x^2 - 3x + 1}{x}\right)\left(\frac{x^2 - 5x + 1}{x}\right) = 6$$

$$\left(x - 3 + \frac{1}{x}\right)\left(x - 5 + \frac{1}{x}\right) = 6$$

$(y - 3)(y - 5) = 6$

$y^2 - 8y + 15 = 6$

$y^2 - 8y + 9 = 0.$

17. Ans: $\log_5 3$ and $\log_5 4$.

Sol: $\left(5^x - 1\right)^2 = 5\left(5^x - \frac{11}{5}\right)$

$= 5^{x+1} - 11$

$(t - 1)^2 = 5t - 11$

$t^2 - 2t + 1 = 5t - 11$

$t^2 - 7t + 12 = 0$

$t = 3, 4$

$5^x = 3$ or $5^x = 4$.

18. Ans: $n + \frac{1}{2}\left(1 - 3^{-n}\right)$

Sol: $S_1 = \frac{4}{3}$

$S_2 = \frac{4}{3} + \frac{10}{9} = \frac{22}{9}$

In option (E) put n = 1, $1 + \frac{1}{2}\left(1 - \frac{1}{3}\right)$

$= 1 + \frac{1}{2} \times \frac{2}{3} = \frac{4}{3}$

Put n = 2, $2 + \frac{1}{2}\left(1 - \frac{1}{9}\right)$

$= 2 + \frac{1}{2} \times \frac{8}{9} = 2 + \frac{4}{9}.$

19. Ans: $-\frac{1}{2}$

Sol: $$\sum_{k=1}^{n}\left(n^3 - n\right) = \left[\frac{n(n+1)}{2}\right]^2 - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} - 1\right]$$

$$= \frac{n+1}{2}\left[\frac{n^3 + n^2 - 2n}{2}\right]$$

S = coefficient of n = $\frac{1}{2}\left(\frac{-2}{2}\right) = \frac{-1}{2}.$

20. Ans: -13

Sol: $\frac{5}{2}[2 \times 2 + 4d] = 5$

$\Rightarrow \frac{5}{2}[4 + 4d] = 5$

$4d = -2$

$\Rightarrow d = \frac{-1}{2}$

$a_{31} = a + 30d = 2 + 30 \times \frac{-1}{2}$

$= 2 - 15$

$= -13.$

21. Ans: $(a + b)^2 + 2(b + c)^2 + (c + d)^2$

Sol: a, b, c, d are in G.P

$\Rightarrow b^2 = ac \qquad c^2 = bd$

$(a + b + c + d)^2$

$= (a + b)^2 + (c + d)^2 + 2(a + b)(c + d)$

$[2(a + b)(c + d) = 2a^2(1 + r)(r^2 + r^3)$

$= 2a^2(r^2 + r^3 + r^3 + r^4)$

$2a^2(r^2 + r)^2 = 2(ar + ar^2)^2$

$= 2(b + c)^2]$

22. Ans: $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y} - x\left(\frac{1-x^ny^n}{1-xy}\right)\right]$

Sol: $1 + (1 + x)y + \left(\frac{1-x^3}{1-x}\right)y^2 + \ldots.$

$= \frac{1-x}{1-x} + \frac{1-x^2}{1-x}y + \frac{1-x^3}{1-x}y^2 + \ldots$

$= \frac{1}{1-x}\left[1 - x + \left(1+x^2\right)y + \left(1-x^3\right)y^2 + \ldots\right]$

$= \frac{1}{1-x}[1 + y + y^2 + \ldots\ldots - x\left[1 + xy + x^2y^2 + \ldots\right]]$

$= \frac{1}{1-x}\left[\frac{1-y^n}{1-y} - x\left(\frac{1-x^ny^n}{1-xy}\right)\right].$

23. Ans: $\frac{5}{4}$

Sol: $x + 2d = a$ ------(1)

$x + 6d = ar$ ------(2)

$x + 11d = ar^2$ ------(3)

$(2) - (1) \Rightarrow 4d = a(r - 1)$

$(3) - (2) \Rightarrow 5d = ar(r - 1)$

Dividing $\frac{5}{4} = r$

24. Ans: 1 . 3. 5 ….(2n – 1)

Sol: $A = \frac{1}{2^n}\left({}^{2n}P_n\right) = \frac{1}{2^n}.\frac{(2n)!}{n!}$

n = 1, A = 1

n = 2, A = 3

n = 3, A = 15

From option (C) 1 . 3. 4 ….. 2n – 1

25. Ans: $\left(1+\frac{1}{n}\right)^n > 2$, n is a positive integer.

Sol: put n = 1

$\left(1+\frac{1}{1}\right)^1 = 2$

$\therefore \left(1+\frac{1}{n}\right) > 2$, n is a positive integer.

26. Ans: 5

Sol: $4(9 - 1)^{501} = 4[M(9) - 1]$

$= M(9) - 4$

Remainder = 5.

27. Ans: 15

Sol: $T_{r+1}\ {}^6C_r\, x^r \left(\frac{1}{x^2}\right)^{6-r}$

$-12 + 3r = 0$

$r = 4$

$\therefore {}^6C_4 = \frac{6\times5}{2} = 15.$

28. Ans: $\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}$

Sol: $(1 + x)^n x^2 = C_0x^2 + \ldots\ldots + C_nx^{n+2}$

Integrating from –1 to 0

$\int_{-1}^{0}(1+x)^n x^2 dx = \frac{C_0}{3} - \frac{C_4}{4} + \ldots..,$

LHS $= \int_{1}^{0}(-x)^n(-1-x)^2 dx$

$= \int_{0}^{1} x^n(x-1)^2 dx.$

29. Ans: 175

Sol:

L	M	
1	3	${}^3C_1 \times {}^7C_3$
2	2	${}^3C_2 \times {}^7C_2$
3	1	${}^3C_3 \times {}^7C_1$

Sum = 175.

30. Ans: 1

Sol: put x = 1

$(1+3-3)^{1/43} = 1.$

31. Ans: 3^{10}

Sol: $\left[1-\left(x^2-4x+4\right)\right]^{10}$

$= (-x^2 + 4x - 3)^{10}$

$= (x^2 + 4x - 3)^{10}$

Constant term $= (-3)^{10} = 3^{10}$.

32. Ans: $(-1, -1)$

Sol: $\begin{bmatrix} e^{x+1} & e^{y+1} \\ e^{y+1} & e^{x+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$x + 1 = 0 \quad y + 1 = 0$

$x = -1 \quad y = -1$

33. Ans: e^2

Sol: $2\log x - \log x = 2$

$\log x = 2$

$x = e^2$.

34. Ans: $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

Sol: $\sin^4\alpha - \cos^4\alpha = 0$

$sn^4\ \alpha = \cos^4\alpha$

$\sin\alpha = \cos\alpha$

$\Rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$.

35. Ans: $(a^2 - 1)^2$

Sol: $|A\ A^T| = |A|\ |A^T|$

$= |A|^2$

$= (a^2 - xy)^2$

$= (a^2 - 1)^2$.

36. Ans: $x^2 (\lambda^2 - 1)$

Sol: $f(x) = x^2 - 2\lambda^2$

$f\lambda x) = \lambda^2 x^2 - 2\lambda^2$

$f(\lambda x) - f(x) = \lambda^2 x^2 - 2\lambda^2 - x^2 + 2\lambda^2$

$= x^2 (\lambda^2 - 1)$.

37. Ans: $q = m$

Sol: $[A]_{m\times n}, [B]_{n\times p}, [C]_{p\times q}$

$[BC]_{n\times q} \quad [A]_{m\times n}$

(BC)A is possible only if $q = m$

38. Ans: $\left(\frac{11}{16}, \infty\right)$

Sol: $\frac{2x+3}{5} < \frac{4x-1}{2}$

$4x + 6 < 20x - 5$

$11 < 16x$

$x > \frac{11}{16} \Rightarrow x \in \left(\frac{11}{16}, \infty\right)$.

39. Ans: $\left(-\frac{3}{2}, \frac{3}{5}\right)$

Sol: $7x - 2 < 4 - 3x$ and $3x - 1 < 2 + 5x$

$10\ x < 6$ and $-3 < 2x$

$x < \frac{3}{5}$ and $x > \frac{-3}{2}$

$x \in \left(\frac{-3}{2}, \frac{3}{5}\right)$.

40. Ans: 1

Sol:

$\sqrt{2}[\cos 45 \cos 30 + \sin 45 + \sin 30 -$

$(\sin 45 \cos 30 - \cos 45 \sin 30)]$

$\sqrt{2}\left[\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right] = \frac{1}{2} + \frac{1}{2} = 1$.

41. Ans: $p \wedge (\sim q)$

Sol: $p \wedge (\sim q)$

42. Ans: Is the sky blue?

Sol: Is the sky blue?

43. Ans: It is not that the earth is round or it is not that 3 + 4 = 7

Sol: It is not that the earth is round or it is not that 3 + 4 = 7

44. Ans: $\frac{1}{\sqrt{10}}$

Sol: $2\cos^2\left(\frac{x}{2}\right) = 1 + \cos x$

$= 1 - \frac{4}{5} = \frac{1}{5}$

$\cos^2 \frac{x}{2} = \frac{1}{10}$,

$\cos \frac{x}{2} = \frac{1}{\sqrt{10}}$ is 1st quadrant.

45. Ans: $\sec^2\theta\ \text{cosec}^2\theta$

Sol: $\sec^2\theta + \text{cosec}^2\theta = \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta \sin^2\theta}$

$= \sec^2\theta\ \text{cosec}^2\theta$.

46. Ans: $(\text{cosec}\ x - \cot x)^2$

Sol: $\frac{1 - \cos x}{1 + \cos x} = \left(\frac{1 - \cos x}{\sin x}\right)^2$

$= (\text{cosec}\ x - \cot x)^2$

47. Ans: 0

Sol: $\sec \frac{2\pi}{3} = \sec(120) = -2 + 2$

48. Ans: $\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

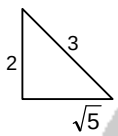
Sol: $\tan^{-1}\left\{\frac{\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}\right)}{1-\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}}\right\}=\tan^{-1}\left(\frac{\frac{3+2}{2\sqrt{3}}}{1-\frac{1}{2}}\right)$

$=\tan^{-1}\frac{5/2\sqrt{3}}{1/2}$

$=\tan^{-1}\left(\frac{5}{\sqrt{3}}\right).$

49. Ans: $\frac{\sqrt{5}}{3}$

Sol:



$\cos\left(\sin^{-1}\frac{2}{3}\right)=\cos\left[\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

$=\frac{\sqrt{5}}{3}$

50. Ans: $\frac{1}{\sqrt{6}}$

Sol: $\tan^{-1}\frac{2x+3x}{1-2x3x}=\frac{\pi}{2}$

$\frac{5x}{1-6x^2}=\tan\frac{\pi}{2}$

$\Rightarrow 1-6x^2=0$

$x^2=\frac{1}{6}$

$x=\frac{1}{\sqrt{6}}.$

51. Ans: $\frac{\pi}{3}$

Sol: $\tan^4 x = 9$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x=\frac{\pi}{3}.$

52. Ans: $\frac{5\pi}{6}$

Sol: $\sec x = -\frac{2}{\sqrt{3}}$

$\cos x = \frac{-\sqrt{3}}{2}$

preferred value of solution $\frac{5\pi}{6}$.

53. Ans: ± 2

Sol: $|-4a - a| = 10$

$-5a = \pm 10$

$a = \pm 2.$

54. Ans: 6

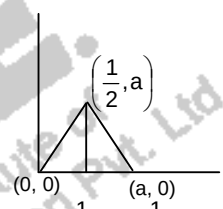
Sol: $\frac{a-4}{-2-3}=\frac{-2}{5}$

$\Rightarrow\frac{a-4}{-5}=\frac{-2}{5}$

$a - 4 = 2,\ a = 6.$

55. Ans: ± 1

Sol:



Area $=\frac{1}{2}a.a=\frac{1}{2}$

$a^2 = 1$

$a = \pm 1.$

56. Ans: $3x + 2y - 6 = 0$

Sol: slope of required line $\frac{-3}{2}$

*** intercept = 3

$y=-\frac{3}{2}x+3$ $\quad 2y + 3x = 6$

$3x + 2y - 6 = 0$

57. Ans: $\sqrt{\frac{13}{2}}$

Sol: $\left|\frac{1+5\times -1-9}{\sqrt{1^2+5^2}}\right|$

$\left|\frac{1-5-9}{\sqrt{26}}\right|=\frac{-13}{\sqrt{26}}=\sqrt{\frac{13}{2}}$

58. Ans: $\tan^{-1}\left(\frac{1}{7}\right)$

Sol: $\tan^{-1}\dfrac{\frac{-2}{11}+\frac{1}{3}}{1+\frac{2}{11}\times\frac{1}{3}} = \tan\left(\frac{-6+11}{33+2}\right)$

$= \tan^{-1}\left(\frac{5}{35}\right) = \tan^{-1}\left(\frac{1}{7}\right).$

59. Ans: 2

Sol: $\dfrac{|2-(-14)|}{\sqrt{5^2+12^2}} = \dfrac{26}{13} = 2$

60. Ans: $x^2 + y^2 = 5$

Sol: $1+ 4 = k^2$

$= 5$

61. Ans: $x^2 + y^2 - 2x - 2y - 2 = 0$

Sol: $r = \dfrac{3\times1+4\times1+3}{\sqrt{3^2+4^2}}$

$= \dfrac{10}{5} = 2$

$(x - 1)^2 + (y - 1)^2 = 4$

62. Ans: (2, 0)

Sol: $x^2 + ax + y^2 = 0$

$16 + 4a = 0$

$4a = -16$

$a = -4$

$x^2 + y^2 - 4x = 0$

63. Ans: $x^2 + y^2 + 4x - 2y + 4 = 0$

Sol: $r = \dfrac{\sqrt{(k+2)^2+4}}{2} = 1$

$(k + 2)^2 + 4 = 4$

$\Rightarrow k = -2$

$\therefore$ Points are $(-2, 2)$, $(-2, 0)$

$\Rightarrow$ centre: $(-2, 1)$, $r = 1$

$(x + 2)^2 + (y - 1)^2 = 1$

$x^2 + y^2 + 4x - 2y + 4 = 0$

64. Ans: -12

Sol: Let $y^2 = 4ax$

Let $(x_1, y_1) = (at_1^2, 2at_1)$ and

$(x_2, y_2) = (at_2^2, 2at_2)$. If they are ends of a focal chord then $t_1t_2 = -1$

$x_1x_2 + y_1y_2 = a^2 (t_1t_2)^2 + 4a^2 (t_1t_2)$

$= a^2 (-1)^2 +4a^2(-1) = -3a^2 = -3(2)^2 = -12$

65. Ans: $\sqrt{\frac{5}{12}}$

Sol: $12x^2 + 7y^2 = 84$

$\Rightarrow \dfrac{x^2}{7}+\dfrac{y^2}{12}=1$

$e = \sqrt{1-\frac{7}{12}} = \sqrt{\frac{5}{12}}$

66. Ans: $x^2 - y^2 = 32$

Sol: $e = \sqrt{2}$

$2ae = 16$

$4a^2e^2 = 16 \times 16$

$\Rightarrow a^2 = \dfrac{16\times16}{4\times2} = 32$

$b^2 = a^2 (e^2 -1) = 32 (2 -1) = 32$

$\therefore x^2 - y^2 = 32$

67. Ans: $(\pm2, 0)$

Sol: $a^2 = 9 - c \quad b' = 5 - c$

$a = \sqrt{9-c} \quad 1 - e^2 = \dfrac{b^2}{a^2} = \dfrac{5-c}{9-c}$

$e^2 = 1-\dfrac{5-c}{9-c} = \dfrac{9-c-5+c}{9-c}$ m

$= \dfrac{4}{9-c} \quad e = \dfrac{2}{\sqrt{9-c}}$

foci $(\pm ae, 0)$

$(\pm, 2, 0)$

$(\pm2, 0)$

68. Ans: $\dfrac{5\sqrt{3}}{2}$

Sol: $\bar{d}_1 = 3\hat{i} - 4\hat{j} - \hat{k} \Rightarrow |\bar{d}_1| = \sqrt{9+16+1}$

$= \sqrt{26}$

$\bar{d}_2 = 2\hat{i} + 3\hat{j} - 6\hat{k} \Rightarrow |\bar{d}_2| = \sqrt{4+9+36}$

$= 7$

$a = \sqrt{\left(\frac{d_1}{2}\right)^2+\left(\frac{d_2}{2}\right)^2} = \sqrt{\frac{26}{4}+\frac{49}{4}} = \sqrt{\frac{75}{4}}$

$= \dfrac{5\sqrt{3}}{2}$

69. Ans: $\dfrac{\pi}{2}$

Sol: $(\bar{a}+\bar{b})\bullet(\bar{a}-\bar{b}) = |\bar{a}|^2 - |\bar{b}|^2$

$= (4 +9 + \alpha^2) - (9+ \alpha^2 +4) = 0$

$\therefore$ Angle $= \dfrac{\pi}{2}$

70. Ans: 1

Sol: $\left|\bar{a}+\bar{b}\right|^2 = \left|\bar{a}-\bar{b}\right|^2$

$\Rightarrow \bar{a}\bullet\bar{b} = 0$

$\Rightarrow 2\alpha + 2\beta - 2 = 0$

$\Rightarrow \alpha + \beta = 1$

71. Ans: $\left|\bar{a}\right|$

Sol: $\dfrac{\bar{b}\bullet\bar{a}}{\left|\bar{a}\right|} = 2\dfrac{\bar{a}\bullet\bar{b}}{\left|\bar{b}\right|}$

$\Rightarrow \dfrac{1}{\left|\bar{a}\right|} = \dfrac{2}{\left|\bar{b}\right|} \Rightarrow \left|\bar{b}\right| = 2\left|\bar{a}\right|$

$\Rightarrow \left|\bar{b}\right| - \left|\bar{a}\right| = \left|\bar{a}\right|$

72. Ans: 4

Sol: $\left|\bar{a}\times\bar{b}\right|^2 + \left(\bar{a}\bullet\bar{b}\right)^2$

$= a^2b^2(\sin^2\theta + \cos^2\theta) = a^2b^2$

$= (1+1)\ (1+1) = 4$

73. Ans: $\dfrac{\pi}{3}$

Sol: $\left|\bar{a}-\bar{b}\right|^2 = \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 - 2\bar{a}\bullet\bar{b}$

$\Rightarrow 7 = 1+9 \ - 2\bar{a}\bullet\bar{b}$

$\Rightarrow 2\bar{a}\bullet\bar{b} = 3$

$a\bullet b = \dfrac{3}{2}$

$\Rightarrow 1\times 3\times\cos\theta = \dfrac{3}{2}$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

74. Ans: $\dfrac{7}{\sqrt{3}}.\left(\hat{i}-\hat{j}-\hat{k}\right)$

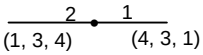
Sol: $\bar{a}+\bar{b} = \hat{i}-\hat{j}-\hat{k}$

$\left|\bar{a}+\bar{b}\right| = \sqrt{3}$

$\therefore$ Required vector $= \dfrac{7.\left(\hat{i}-\hat{j}-\hat{k}\right)}{\sqrt{3}}$

75. Ans: (3, 3, 2)

Sol: Required point



$= \dfrac{2(4,\ 3,\ 1)+1(1,\ 3,\ 4)}{3}$

$= \left(\dfrac{9}{3}, \dfrac{9}{3}, \dfrac{6}{3}\right) = (3,\ 3,\ 2)$

76. Ans: $\dfrac{\pi}{2}$

Sol: The lines are $\dfrac{x-7}{1} = \dfrac{y+3}{-5} = \dfrac{z}{3}$ and

$\dfrac{x-2}{7} = \dfrac{y}{2} = \dfrac{z+5}{1}$

$\cos\theta = \dfrac{7-10+3}{\sqrt{1+25+9}.\sqrt{49+4+1}}$

$\Rightarrow \theta = \dfrac{\pi}{2}$

77. Ans: $8x - 8y + 4z + 15 = 0$

Sol: The two planes are $2x - 2y + z + 3 = 0$ and

$2x - 2y + z + \dfrac{9}{2} = 0$

The required plane is

$2x - 2y + z + \left(\dfrac{3+\frac{9}{2}}{2}\right) = 0$

$\Rightarrow 2x - 2y + z + \dfrac{15}{4} = 0$

$\Rightarrow 8x - 8y + 4z + 15 = 0$

78. Ans: $\dfrac{\pi}{3}$

Sol: $\cos\theta = \dfrac{12-12+25}{\sqrt{9+16+25}.\sqrt{16+9+25}}$

$= \dfrac{25}{50} = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

79. Ans: $\bar{r}\bullet\left(\hat{i}+3\hat{j}-\hat{k}\right) = 6$

Sol: The required plane is $x + 3y - z + k = 0$
which passes through (2, 1, –1)

$\Rightarrow 2 + 3 + 1 + k = 0$

$\Rightarrow k = -6$

$\Rightarrow x + 3y - z - 6 = 0$

$\Rightarrow \bar{r}\bullet\left(\hat{i}+3\hat{j}-\hat{k}\right) = 6$

80. Ans: $2x + y - 3z + 8 = 0$

Sol: DR : 4, 2, –6
i.e, 2, 1, –3
$\therefore$ Required plane is
$2(x - 1) + 1(y + 1) - 3(z - 3) = 0$
$\Rightarrow 2x + y - 3z + 8 = 0$

81. Ans: $2y + z = 2$

Sol: The required plane is
$x - y + z + 3 + \lambda(x + y + 2z + 1) = 0$
where x term is absent
$\therefore$ equation is $2y + z = 2$

82. Ans: $\sqrt{2}\ (x-2) = \sqrt{2}\ (y-1) = z+1$

Sol: The d.r.' s are 1, 1, $\sqrt{2}$

$\therefore$ line is $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z+1}{\sqrt{2}}$

i.e. $\sqrt{2}\ (x-2) = \sqrt{2}\ (y-1) = z+1$

83. Ans: $\frac{7}{16}$

Sol: Possible pairs (2, 1), (3, 1), (3, 2), (4, 1)
(4, 2), (4, 3), (4, 4) (7)
No. of possible pairs = $4 \times 4 = 16$

Required probability = $\frac{7}{16}$

84. Ans: $\frac{5}{108}$

Sol: x + y + z = 15
x, y, z $\in$ {1, 2, 3, 4, 5, 6}
No. of possibilities = 10
The favourable cases:
(6, 6, 3), (6, 5, 4), (6, 4, 5), (6, 3, 6)
(5, 6, 4), 5, 5, 5), (5, 4, 6)
(4, 6, 5), (4, 5, 6), (3, 6, 6)

Probability = $\frac{10}{216} = \frac{5}{108}$

85. Ans: {–2, 0}

Sol: $\frac{\sum X^2}{5} = 2 \qquad \frac{\sum X}{5} = 0$

$\therefore \sum X = 0$

The sum of the 2 Nos. = –2
Sum of squares of the two Nos. = 4

86. Ans: 40

Sol: $15\ (\bar{x}) + 70 = 16(\bar{x}+2)$

$\Rightarrow \bar{x} = 38$

New mean = $\bar{x} + 2 = 40$.

87. Ans: 2

Sol: $\lim_{x\to 0} \frac{\log(1+2x)}{x} = \lim_{2x\to 0} \frac{\log(1+2x)}{2x}.2$

$= 2$

88. Ans: $\frac{1}{3}$

Sol: $\lim_{x\to 0^-} \frac{1}{3-2^{1/x}} = \lim_{y\to 0^+} \frac{1}{3-2^{-1/y}}$

$= \frac{1}{3-0} = \frac{1}{3}$

89. Ans: 2x – 5

Sol: f(x) = 2(x – 2) – (x + 1) + x (x $\geq$ 2)
= 2x – 4 – x – 1 + x
= 2x – 5

90. Ans: –3, 4

Sol: 1 = c + k
4c + k = –8 $\Rightarrow$ k = 4, c = –3

91. Ans: 10 log5

Sol: $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 5^{\tan\frac{\pi}{4}} \sec^2\frac{\pi}{4}.\log 5$

= 10 log5

92. Ans: $\frac{|x|}{x}(=1)$

Sol: $\frac{dy}{dz} = \frac{1}{\sqrt{1-x^2}} \div \frac{-1}{\sqrt{1-(1-x^2)}}.\frac{1}{2\sqrt{1-x^2}}.-2x$

$= \frac{1}{\sqrt{1-x^2}} \frac{|x|\sqrt{1-x^2}}{x} = \frac{|x|}{x}$

The most appropriate answer is 1.

93. Ans: $\frac{1}{\sqrt{3}}$

Sol: $\frac{dv}{du} = \frac{2\sin t}{2(1-\cos t)} = \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{1}{\sqrt{3}}$

94. Ans: $\frac{3}{4}$

Sol: f (x) = x + $\frac{1}{3}$ [log(3 – x) – log (3+x)]

f'(x) = 1 – $\frac{1}{3}\left[\frac{1}{3-x} + \frac{1}{3+x}\right]$

f'(1) = 1 – $\frac{1}{3}\left[\frac{1}{2} + \frac{1}{4}\right]$

= 1 – $\frac{1}{4} = \frac{3}{4}$

95. Ans: $\frac{2}{e}$

Sol: $\frac{dy}{dx} = \frac{2\log x}{e} = \frac{2}{e}$, at $x = e$

96. Ans: $\frac{y}{x}\log\left(\frac{2}{y}\right)$

Sol: x log y = x log 2

$\log y + \frac{x}{y}\frac{dy}{dx} = \log 2$

$\frac{dy}{dx} = \frac{y}{x}\log\frac{2}{y}$

97. Ans: 0

Sol: When y = 1, x = –1

$$2x+2\left(x\frac{dy}{dx}+y\right)+4y\frac{dy}{dx}=0$$

$\therefore$ at y = 1,

$$-2+2\left(-\frac{dy}{dx}\right)+2+4\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=0 \quad \text{at } y=1$$

98. Ans: (1, –2), (–1, 2)

Sol: $\frac{dy}{dx}=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

When $x=\pm 1$, $y=\mp 2$

Points are (1, –2) and (–1, 2)

99. Ans: –6

Sol: $(3y^2-x)\frac{dy}{dx}-y=0$ at (0, 2)

$$12\frac{dy}{dx}-2=0$$

$$\frac{dy}{dx}=\frac{1}{6}$$

$\therefore$ slope of normal = –6

100. Ans: 1, and – 2

Sol: At x = 1, y = 1

$$\frac{xdy}{dx}+y+a+b\frac{dy}{dx}=0$$

$\therefore$ 2 + 1 + a + 2b = 0

and a + b + 1 = 0

b = –2 a = 1

101. Ans: $\dfrac{\log\frac{3}{2}}{1+\log 2\log 3}$

Sol: $y=2^x \Rightarrow \frac{dy}{dx}=2^x\log 2$

$y=3^x \Rightarrow \frac{dy}{dx}=3^x\log 3$

At intersection $2^X=3^x \Rightarrow x=0$

$\therefore$ slopes at intersection are log2, log3

If acute angle is α,

$$\tan\alpha=\frac{\log 3-\log 2}{1+\log 2\log 3}=\frac{\log\frac{3}{2}}{1+(\log 2)\log 3}$$

$$\frac{\log\frac{3}{2}}{1+\log 2\log 3}$$

102. Ans: x < – 2 or > 2

Sol: $f(x)=9x^2-36$

f is increasing if $x^2>4$

x < – 2 or > 2

103. Ans: $\frac{2\sqrt{3}}{3}$

Sol: $3c^2-1=\frac{6+p-p}{2}$

$$c^2=\frac{4}{3}$$

$$c=\pm\frac{2}{\sqrt{3}}\ \&\ c\in(0,2)$$

$$\therefore c=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$

104. Ans: $\frac{\sqrt{2}}{2}$

Sol: $f(x)=\dfrac{1}{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)}$

$\therefore$ minimum Value $=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

105. Ans: $+\frac{5}{2(x-1)^2}+\frac{5}{x-1}+C$

Sol: $\int\frac{5xdy}{(1-x)^3}=\int\frac{5(x-1)+5}{(1-x)^3}dx$

$$=\int\left(-\frac{5}{(1-x)^2}+\frac{5}{(1-x)^3}\right)dx$$

$$=-\frac{5}{1-x}+\frac{5}{2(1-x)^2}+C$$

$$=+\frac{5}{2(x-1)^2}+\frac{5}{x-1}+C$$

106. Ans: $2\log\left|\sqrt{x}-1\right|+C$

Sol: $\int\frac{dx}{\sqrt{x}\left(\sqrt{x}-1\right)}=2\int\frac{dx}{\frac{2\sqrt{x}}{\sqrt{x}-1}}$

$$2\log\left|\sqrt{x}-1\right|+C$$

107. Ans: $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}+C\right)$

Sol: $\int\frac{\sec^2 xdy}{3+4\tan^2 x}$

$$=\frac{1}{2}\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}+C\right)$$

108. Ans: $\sin^{-1}(\tan x) + C$

Sol: $\int \frac{\sec x\, dx}{\sqrt{\frac{1-\tan^2 x}{1+\tan^2 x}}} = \int \frac{\sec^2 x\, dx}{\sqrt{1-\tan^2 x}}$

$= \sin^{-1}(\tan x) + C$

109. Ans: $e^x \log|x| + C$

Sol: $\int e^x\left(\log + \frac{1}{x}\right)dx = e^x \log|x| + C.$

110. Ans: $\frac{-1}{1+x\log|x|} + C$

Sol: $\int \frac{(1+\log x)dx}{(1+x\log x)^2} = \int \frac{du}{u^2}$

where $u = 1 + x\log x$

$= C - \frac{1}{1+x\log|x|}$.

111. Ans: $2x - \tan x + c$

Sol: $\int (1-\tan^2 x)dx = \int (2-\sec^2 x)dx$

$= 2x - \tan x + c$

112. Ans: 9

Sol: $= \int_0^3 (3-x)dx + \int_3^6 (x-3)dx$

$= \left.\frac{(3-x)^2}{2}\right|_3^0 + \left.\frac{(x-3)^2}{2}\right|_3^6$

$= \frac{9}{2} + \frac{9}{2} = 9$

113. Ans: $\cos(\sin^3 x)\cos x$ — $2\cos(8x^3)$.

Sol: $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)dt = f(\psi(x))\psi'(x)$

$-f(\phi(x))\,\phi'(x)$

Thus required derivative

$= \cos(\sin^3 x)\cos x - \cos(8x^3).2$

$= \cos(\sin^3 x)\cos x - 2\cos(8x^3)$.

114. Ans: 5

Sol: $I = \int_0^{10} \frac{x^{10}dx}{(10-x)^{10}+x^{10}} \int_0^{10} \frac{(10-x)^{10}dx}{x^{10}+(10-x)^{10}}$

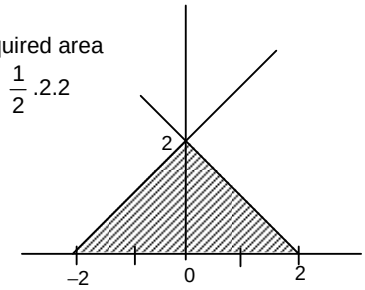
$= \frac{1}{2}\int_0^{10} \frac{x^{10}+(10-x)^{10}}{(10-x)^{10}+x^{10}}dx = \frac{1}{2}.10 = 5$

115. Ans: 4

Sol: Required area

$= 2.\ \frac{1}{2}.2.2$

$= 4$



116. Ans: $2(e-2)$

Sol: $\int_0^1 \sqrt{x}\ e^{\sqrt{x}}\ dx = 2\int_0^1 x^2 e^x dx$

$= \left[2[x^2 - 2x + 2]e^x\right]_0^1$

$= 2\,[e\,(1 - 2+2) - 2]$

$= 2\,(e - 2)$

117. Ans: 3, 1

Sol: Order = 3, degree = 1

118. Ans: $y'' + 3y' = 0$

Sol: $y = ae^{-3x} + b$

$y' = -3ae^{-3x}$

$y'' = 9ae^{-3x} = -3y'$

i.e. $y'' + 3y' = 0$

119. Ans: $\frac{1}{x}$

Sol: Multiply by $\frac{1}{x}$

Equation becomes

$\frac{dy}{dx} - \frac{y}{x} = -xe^x$

I.F of this equation $= e^{-\log x} = \frac{1}{x}$

$\therefore$ I.F of given equation is $\frac{1}{x^2}$

But the transformed equation has IF $\frac{1}{x}$

120. Ans: $y = C(1 + \log x)$

Sol: $\frac{dy}{y} = \frac{dx}{x(1+\log x)}$

$\therefore \log|y| = \log|(1 + \log x)|$

$y = C(1 + \log x)$